

## Math 174 Numerical Methods in Science and Engineering Fall 2004

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<b>Office Hours:</b>	M 2:10p-3:00p, Tu 2:00p-3:00p, or by appointment
<b>Meeting Times:</b>	MWF 4:00p-4:50p
<b>Room:</b>	1315 Humanities and Social Sciences Building (HSS)
<b>Textbook:</b>	Numerical Methods Course Notes, UCSD M174. Available online from the course webpage or at Soft Reserves.
<b>Prerequisites:</b>	Math 21D and 20F
<b>Course Webpage:</b>	<a href="http://scicomp.ucsd.edu/~spav/class/2004F-M174/">http://scicomp.ucsd.edu/~spav/class/2004F-M174/</a>
<b>Final Exam:</b>	Thursday December 9, 3:00p-6:00p
<b>Note:</b>	Discussion Session meets Wednesdays 7:00pm-7:50pm, 1315 HSS.

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**Catalog Description.** *174. Numerical Methods in Science and Engineering (4)* Floating point arithmetic, linear equations, interpolation, integration, differential equations, nonlinear equations, optimization, least squares. Students may not receive credit for both Math. 174 and Physics 105 or MAE 153 or 154. Students may not receive credit for Math. 174 if Math. 170 A,B, or C has already been taken. *Prerequisites: Math. 21D (2DA) and Math. 20F (2EA).*

**Course Description.** This course serves as an introduction to applied numerical mathematics. Numerical analysis, roughly speaking, is concerned with finding approximate numerical solutions to problems for which we lack sufficient data or have no analytic solution: for example, we will explore methods of finding approximate roots of functions for which we have no closed form solution; we will examine algorithms which approximate the integral of a “black box” function; we will use iterative solvers to find the approximate solutions to linear systems; we will use the method of least squares to find the “best” function to approximate a set of data points.

**Grading Policy.** Grading will be based upon performance in eight homeworks, three quizzes, two single hour exams and a comprehensive final. The final quarter grade is subdivided as follows: Homework: 15%; Quizzes: 15%; Exams: 35%; Final: 35%.

Homeworks will be collected in the discussion section. The TA has discretion regarding accepting homework by proxy or via his/her mailbox. It is my expectation that the TA will cover homework during the discussion section, allowing students to make some changes to their homework before submitting same. It is also my expectation that the TA will not accept late homework. Students are encouraged to work together on homeworks (but prohibited from doing so on quizzes and exams). *However*, each student must hand in their own homework, with their own name. Moreover, students should list on their homework who they worked with.

Quizzes will be administered in the discussion section; the material on the quizzes will be similar to that of the assigned homeworks. The purpose of the quizzes is not to make you miserable, rather to make you review the material between the exams. The first quiz will be returned before the cutoff date for ‘drop without a W,’ *i.e.*, the end of the fourth week.

The midterm exams will be administered in class, on Monday, October 25, and Friday, November 19. The final is Thursday, December 6 at 3:00pm. If you have known conflicts with any of these exams, *you must alert me of this before Friday October 1.* Legitimate, documented excuses for missing an exam will be dealt with individually.

The two midterm exams account for 35% of the final grade. Your better score of the two

midterms will be worth 20% of your grade, while your lesser score will be worth 15%. This is supposed to skew scores upwards, and decrease the effects of a “bad day.”

Final scores will be converted into letter grades as follows:

Final Score	Final Grade
at least 90%	A- or better
at least 80%	B- or better
at least 70%	C- or better
at least 60%	D or better

These grade “promises” are guaranteed, but note that the “or better” might be *much* better, depending on certain factors: if the exams are too difficult, or the grading too harsh, etc. Thus, for example, the cutoff for a C- might be as low as 65%, say.

Students are expected to adhere to the University’s policy on academic integrity.

**Course Webpage.** The course webpage, (<http://scicomp.ucsd.edu/~spav/class/2004F-M174/>) will include any relevant notices and news. The course textbook is also available from the webpage.

**Course Schedule.** The lecture schedule is tentative, but the homework and exam schedules are exact. Section numbers refer to the course notes.

week 0	F Sep 24	Syllabus & Taylor’s Theorem §1.1
week 1	M Sep 27	Subtractive cancellation, Linear Algebra Review §1.2,1.3
	W Sep 29	Introduction to octave/Matlab §2.1-2.4
	W Sep 29 (disc.)	octave/Matlab demo?
	F Oct 01	Linear Equations §3.1
week 2	M Oct 04	Linear Equations §3.1,3.2
	W Oct 06	Linear Equations §3.2,3.3
	W Oct 06 (disc.)	HW 1 : §1 # 1,2,4,5,8,9,11,12,13,14 §2 # 1,2,3,4
	F Oct 08	Linear Equations §3.4
week 3	M Oct 11	Linear Equations §3.4
	W Oct 13	Rootfinding §4.1
	W Oct 13 (disc.)	HW 2 : §1 # 15,16,17,19,21,22,23 §3 # 1,2,4,5,7 quiz 1 covering §1.1–1.3,3.1–3.4
	F Oct 15	Rootfinding §4.2
week 4	M Oct 18	Rootfinding §4.3
	W Oct 20	Interpolation §5.1
	W Oct 20 (disc.)	HW 3 : §3 # 8 §4 # 1,2,3,4,6,7
	F Oct 22	Interpolation §5.2
week 5	M Oct 25	<b>exam 1 covering §1.1–1.3,3.1–3.4,4.1–4.3</b>
	W Oct 27	Spline Interpolation §6.1, 6.2
	W Oct 27 (disc.)	HW 4 : §4 # 8 §5 # 1,2,3,4,5,6,7,8,10
	F Oct 29	Spline Interpolation §6.3

week 6	M Nov 01	Derivatives §7.1
	W Nov 03	Derivatives §7.2
	W Nov 03 (disc.)	<div style="border: 1px solid black; padding: 5px;">           §5 # 11            HW 5 : §6 1,2,3,4,5,6,7,8            §7 # 1,2,3         </div>
	F Nov 05	Quadrature §8.1
week 7	M Nov 08	Quadrature §8.2
	W Nov 10	Quadrature §8.3
	W Nov 10 (disc.)	<div style="border: 1px solid black; padding: 5px;">           HW 6 : §7 # 4,5,6            §8 # 1,2,3,4,5         </div> <div style="border: 1px solid black; padding: 5px; margin-top: 5px;">           quiz 2 covering §5.1,5.2,6.1–6.3,7.1,7.2,8.1         </div>
	F Nov 12	Quadrature §8.4
week 8	M Nov 15	Least Squares §9.1
	W Nov 17	Least Squares §9.2
	W Nov 17 (disc.)	<div style="border: 1px solid black; padding: 5px;">           HW 7 : §8 # 6,7,9,10            §9 # 1,2,3,4,5         </div>
F Nov 19	<b>exam 2 covering §5.1,5.2,6.1–6.3,7.1,7.2,8.1–8.4</b>	
week 9	M Nov 22	ODEs §10.1
	W Nov 24	ODEs §10.1,10.2
	W Nov 24 (disc.)	
F Nov 26	Thanksgiving–No Meeting	
week 10	M Nov 29	ODEs §10.3
	W Dec 01	ODEs §10.3
	W Dec 01 (disc.)	<div style="border: 1px solid black; padding: 5px;">           HW 8 : §9 # 6            §10 # 1,2,3,4,5,6         </div> <div style="border: 1px solid black; padding: 5px; margin-top: 5px;">           quiz 3 covering §9.1,9.2,10.1–10.2         </div>
	F Dec 03	final exam review
finals	Th Dec 9	<b>Comprehensive Final Exam. 3:00p-6:00p</b>

**Sample Exam Questions.** What follows are a number of questions which might appear on the midterm exams or final exam. If you can easily answer them at start of semester, you may not need to take this class;

1. State Taylor's theorem.
2. Find a way of computing  $\sqrt{x^4 + 4} - 2$  for  $x$  near 0 without incurring subtraction error.
3. Use Newton's method to demonstrate an algorithm which can solve  $x = 1/R$ , for a given  $R$ , *without using division*.
4. Write out the Lagrange Polynomial  $\ell_0(x)$  for nodes  $x_0, x_1, \dots, x_n$ . That is, write the polynomial that has value 1 at  $x_0$  and has value 0 at  $x_1, x_2, \dots, x_n$ .
5. Find the polynomial of least degree that interpolates the following data:

$x$	1	2	3	7
$y$	2	1	12	146

6. Determine the error term of the approximation:

$$f''(x) \approx \frac{1}{h^2} [f(x-h) - 2f(x) + f(x+h)]$$

7. A function,  $f(x)$  is said to be *convex* if its graph lies beneath every chord drawn between two points of a graph; for example  $x^2$  is convex. What is the relationship between  $\int_a^b f(x)dx$  and the three approximations: Left-hand, Right-hand, and Trapezoid, for convex  $f(x)$ ?
8. Write the Trapezoid rule approximation for  $\int_a^b f(x)dx$ , using the nodes  $\{x_i\}_{i=0}^n$ .
9. Determine values of  $\alpha, \beta, \gamma$  such that the quadrature rule

$$\int_{-1}^1 f(x)dx \approx \alpha f(-0.5) + \beta f(0) + \gamma f(0.5)$$

is exact for all polynomial  $f(x)$  of degree  $\leq 2$ .

10. Use naïve Gaussian elimination to solve the linear system:

$$\begin{aligned} 2x + 4y - 2z &= 2 \\ 4x + 9y - 3z &= 8 \\ -2x - 3y + 7z &= 10 \end{aligned}$$

11. Determine the natural cubic spline interpolating the function  $f(x) = x^6$  over the interval  $[0, 2]$  using knots 0, 1, 2.
12. Find the parabola  $f(x) = ax^2 + b$  that best represents (in the Least-Squares sense) the following data

$x$	-1	0	1
$y$	3.1	0.9	2.9

13. Consider the ODE:

$$\begin{cases} x'(t) = f(t, x(t)) \\ x(a) = c \end{cases}$$

Write out the Euler Method to step from  $x(t)$  to  $x(t+h)$ .