Final Exam Preparation

The final exam is Thursday December 9, 3:00pm-6:00pm, in the regular class room. *You must bring a blue book to the exam.* The final exam is comprehensive: it covers all sections of the notes, except the material on Matlab, chapter 2. This prep sheet only covers material which we've studied since the last midterm exam. To prepare for earlier material, see the prep sheets for the previous exams.

You should be prepared to answer at least the following questions:

1. (§9.1) Find the function $f(x) = c$ which best approximates, in the least squares sense, the data

x	1	-2	5
y	1	-2	4

2. (§9.1) Set up the normal equations to find the function $f(x) = c_0 + c_1 x + c_2 x^2$ which best approximates, in the least squares sense, the data

x_0	x_1	$\dots$	x_n
y_0	y_1	$\dots$	y_n

Your equations should have the constants c_0, c_1, c_2 as unknowns.

3. (§9.1) Suppose you want to find a function $f(x) = cx^k$ that "best" approximates a set of data

x_0	x_1	$\dots$	x_n
y_0	y_1	$\dots$	y_n

where both constants c and k are to be determined. Can you think of a way to do this? Will it give the least-squares best approximation?

4. (§10.1) Consider the separable ODE

$$x'(t) = x(t)g(t),$$

for some function g. Use backward's Euler (or, alternatively, the right endpoint rule for approximating integrals) to derive the approximation scheme

$$x(t+h) \leftarrow \frac{x(t)}{1 - hg(t+h)}.$$

What could go wrong with using such a scheme?

5. (§10.1) Consider the ODE

$$x'(t) = x(t) + t^2.$$

Using Taylor's Theorem, derive a higher order approximation scheme to find $x(t+h)$ based on $x(t)$, t, and h.

6. (§10.1) Which of the following ODEs are stable or unstable? For which of them is our stability test ambiguous?

(a) $x'(t) = x + t \arctan x$,
(b) $x'(t) = 4x - e^x$,
(c) $x'(t) = \cos x + \sin x - t$,
(d) $x'(t) = 4x^2$.

7. (§10.2) Consider the ODE

$$x'(t) = \frac{t}{1 - x(t)}.$$

Let $x(0) = 0$. Approximate $x(0.2)$ using one step of Euler's Method. Do the same using Runge-Kutta 2.

8. (§10.3) Rewrite the system of ODEs as a system of first order ODEs

$$\begin{cases} x''(t) = y(t) + t - x(t), \\ y'(t) = x'(t) + x(t) - 4, \\ x'(0) = 1, \\ x(0) = 2, \\ y(0) = 0. \end{cases}$$

9. (§10.3) Consider the system of ODEs

$$\begin{cases} x'(t) = y(t)x(t), \\ y'(t) = x(t)/y(t), \\ x(0) = 1, \\ y(0) = 2. \end{cases}$$

Approximate $x(0.5)$ and $y(0.5)$ using two steps of Euler's method. Approximate them using two steps of Runge-Kutta 2.