

Name \_\_\_\_\_ Student # \_\_\_\_\_

**Instructions:** Read all instructions carefully. Write your name and student number above. Clearly indicate your answers & show all your work. For many problems partial credit is available. 6 Problems worth 30 Points.

**Problems. Show all work on your answer sheets. Partial credit is available.**

P1 (5 pnts) Clearly state Taylor's Theorem for  $f(x + h)$ . Include all hypotheses, and state the conditions on the variable that appears in the error term.

P2 (5 pnts) Give a  $\mathcal{O}(h^3)$  approximation to  $\ln(1 + h)$ . Suppose that  $h$  is restricted to be in  $[-0.5, 0.5]$  How big could the error be?

P3 (5 pnts) Rewrite the expression  $1 - \cos x$  to eliminate subtractive cancelling when  $x$  is small. (*Hint:* you need not use Taylor's Theorem, as the simpler technique suffices.)

P4 (5 pnts) Find the LU decomposition of the following matrix by Gaussian Elimination with naïve pivoting.

$$\begin{bmatrix} 2 & 4 & 10 \\ 6 & 14 & 20 \\ -4 & -2 & -17 \end{bmatrix}$$

P5 (5 pnts) Suppose the eigenvalues of the  $3 \times 3$  matrix  $A$  are 1, 3, 20. Give an eigenvalue of

$$B = A^2 - 2A - 3I,$$

where  $I$  is the identity matrix. Is the matrix  $B$  singular? Why or why not?

P6 (5 pnts) Write the general form of the iterated solution to the problem

$$A\mathbf{x} = \mathbf{b}.$$

Let  $Q$  be your "splitting matrix," and use the factor  $\omega$ . Your solution should look something like:

$$??\mathbf{x}^{(k)} = ??\mathbf{x}^{(k-1)} + ?? \quad \text{or} \quad \mathbf{x}^{(k)} = ??\mathbf{x}^{(k-1)} + ??$$

For one of the following variants, describe what  $Q$  is, in relation to  $A$  : Richardson's, Jacobi, Gauss-Seidel.