Exam 1 Preparation

The first midterm exam is Monday October 18, during the class period. *You must bring a blue book to the exam.* Blue books are available from the bookstore. You also need to bring your student ID card or other form of ID (driver's license, passport, etc.)

You should be prepared to answer at least the following questions:

1. (§3.11) Approximate $\sqrt{4.1}$ using linearizations.

2. (§3.11) Approximate $\cos(44.8^\circ)$ using linearizations.

3. (§3.11) Find the linearization of $\ln x$ at 1.

4. (§3.11) A sphere is measured to have a radius of 10cm, with an error of ± 1mm. *Estimate* the error in the calculated volume of the sphere.

5. (§4.2) Verify that $f(x) = x^3 + x - 1$ satisfies the hypotheses of the Mean Value Theorem on $[0, 2]$. Find a c in this interval that satisfies the conclusions of that theorem.

6. (§4.2) Can the equation $4 + 3x + x^7 = 0$ have *two* distinct roots. (*Hint:* assume there are two distinct roots, and show a contradiction.)

7. (§4.2) Using the Mean Value Theorem, find upper and lower bounds on $f(4)$ for continuous, differentiable function $f(x)$ with the properties that $f(0) = 1$, and $-3 \le f'(x) \le 1$ for x in $[0, 4]$.

8. (§4.2) Let $f(x) = |x - 2|$. Show there is no c such that $f(4) - f(1) = f'(c)(4 - 1)$. Doesn't this contradict the Mean Value Theorem?

9. (§4.10) Find the most general antiderivative of the following:

$$f(x) = x^3 - 2, \qquad g(x) = \sqrt{2x} + x^{-4}, \qquad h(x) = 4\cos x - \sin x,$$
$$j(x) = 1 + x^{3/5}, \qquad k(x) = 4/(1 + x^2)$$

10. (§5.2) Estimate $\int_0^1 x^2 \mathrm{d}x$ by a Riemann sum, using 4 subintervals, and using the right hand endpoint rule. Is this an underestimate or overestimate of the area?

11. (§5.3) State the Fundamental Theorem of Calculus, parts 1 and 2.

12. (§5.3) Find the derivative of $f(x) = \int_1^x \sqrt{1 + t^3}\mathrm{d}t$.

13. (§5.3) Find the derivative of $f(x) = \int_x^3 \sin\left(\sqrt[4]{t}\right) \mathrm{d}t$.

14. (§5.3) Find the derivative of $f(x) = \int_0^{x^2} \sqrt{1 + t^4}\mathrm{d}t$.

15. (§5.3) Let $f(x) = \int_{1/x}^{1+x^2} t^3 \mathrm{d}t$. Evaluate $f'(2)$. (*Hint:* split the integral as $f(x) = \int_{1/x}^1 t^3\mathrm{d}t + \int_1^{1+x^2} t^3\mathrm{d}t$, and use the Fundamental Theorem twice.)

16. (§5.3) Give a physical interpretation of the definite integral $\int_1^2 x^2\mathrm{d}x$. Make a drawing (graph) to show this interpretation.

17. (§5.3) Evaluate the following definite integrals:

$$\int_2^4 \mathrm{d}x \quad \int_0^1 6x^2\mathrm{d}x \quad \int_\pi^{2\pi} \cos\theta\mathrm{d}\theta \quad \int_0^1 \left(3 + x\sqrt{x}\right)\mathrm{d}x \quad \int_1^2 \frac{3 + x^2}{x^3}\mathrm{d}x \quad \int_1^e t^{-1}\mathrm{d}t$$

18. (§5.4) Evaluate the indefinite integrals: (Don't forget the "$+C$")

$$\int x^n \mathrm{d}x, \text{ with } n \neq -1, \quad \int x^{-1}\mathrm{d}x, \quad \int e^x \mathrm{d}x, \quad \int \sin x \mathrm{d}x, \quad \int \cos x \mathrm{d}x,$$
$$\int 1/(1+x^2)\mathrm{d}x, \quad \int \sec^2 x \mathrm{d}x, \quad \int \sec x \tan x \mathrm{d}x,$$

19. (§5.5) Evaluate the indefinite and definite integrals:

$$\int x^2 \left(x^3-1\right)^6 \mathrm{d}x, \quad \int \frac{x+5x^4}{\sqrt{1+x^2+2x^5}}\mathrm{d}x, \quad \int \frac{\sin x}{1+\cos^2 x}\mathrm{d}x, \quad \int \frac{1}{x\ln x}\mathrm{d}x,$$
$$\int \sqrt{4+3x}\mathrm{d}x, \quad \int e^x \sin\left(1+e^x\right)\mathrm{d}x, \quad \int \frac{e^{1/x}}{x^2}\mathrm{d}x,$$
$$\int_0^2 (x-1)^4\,\mathrm{d}x, \quad \int_0^{\sqrt{\pi}} x\cos x^2 \mathrm{d}x, \quad \int_0^{\pi} \sin x \sin(\cos x)\,\mathrm{d}x, \quad \int_1^{e^\pi} \frac{\cos(\ln x)}{x}\mathrm{d}x,$$

20. (§5.6) Give the definition of $\ln x$.

21. (§5.6) Use the fact that for $t > 1$,

$$\frac{1}{t^2} \leq \frac{1}{t} \leq \frac{1}{\sqrt{t}}$$

to estimate $\ln 2$.

22. (§6.1) Find the area of each of the regions enclosed by the given curves:

1. $y = 4x,\ y = 1/x,\ x = 1,\ x = 4.$
2. $y = x + 2,\ y = 4 - x^2.$
3. $y = 1/x,\ y = 1/x^2,\ x = 3.$
4. $y = x^2,\ x = y^2.$
5. $y = \sin x,\ y = \sin 2x,\ x = 0,\ x = \pi/2.$ (*Hint:* you may wish to use the fact that $\sin 2x = 2\sin x \cos x$.)

23. (§6.2) Let $A(x)$ be the cross sectional area of shape S, which is bounded by the planes $x = a$, $x = b$. What is the volume of S?

24. (§6.2) Find the volume of each of the following shapes:

1. Rotation of region bounded by $y = \sqrt{x}$, $x = 0$, $x = 2$ around the x-axis.
2. Rotation of region bounded by $y = \sqrt{\ln x^{1/x}}$, $x = 1$, $x = e$ around the x-axis.
3. Rotation of region bounded by $y = x^2$, $y = 0$, $y = 4$ around the y-axis.
4. Rotation of region bounded by $y = x^2$, $x = 0$, $x = 2$ around the y-axis.
5. Rotation of region bounded by $y = x^2$, $y = 4$, around the line $y = 4$.
6. Rotation of region bounded by $y = x$, $y = \sqrt{x}$, around the line $x = 2$.

25. (§6.5) Find the average value of the function on the given interval:

1. $f(x) = \sqrt{x}, \quad [0, 2].$
2. $f(x) = x\cos\left(x^2/2\right), \quad [0, \sqrt{\pi}].$

26. (misc.) Give an upper bound on $\int_0^1 \sin\sqrt[4]{t}\mathrm{d}t$. Give a lower bound on this integral using the fact that $\sin x \geq x$ for x in $[0, 1]$.