Final Exam Preparation (comprehensive version)

This preparation sheet is a compilation of all three prep sheets from this quarter.

The second midterm exam is Monday December 6, 11:30am-2:30pm, in the regular class room, WLH 2005. *You must bring a blue book to the exam.* Blue books are available from the bookstore. You also need to bring your student ID card or other form of ID (driver's license, passport, etc.)

The following formulæ will be provided on your exam:

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}, \quad \sin^2\theta = \frac{1-\cos 2\theta}{2},$$
$$\cos 2\theta = \cos^2\theta - \sin^2\theta, \quad \sin 2\theta = 2\sin\theta\cos\theta.$$

Everything else must be committed to memory.

The final exam is comprehensive, covering all the material from this quarter. However, there should be more questions focusing on material we have covered since the second midterm.

You should be prepared to answer questions like those that follow. Note that I have not checked many of these problems, some of them may be unsolveable, impractically difficult or generally insane. You are warned.

Exam 1 Prep Sheet:

1. (§3.11) Approximate $\sqrt{4.1}$ using linearizations.

2. (§3.11) Approximate $\cos(44.8^\circ)$ using linearizations.

3. (§3.11) Find the linearization of $\ln x$ at 1.

4. (§3.11) A sphere is measured to have a radius of 10cm, with an error of ±1mm. *Estimate* the error in the calculated volume of the sphere.

5. (§4.2) Verify that $f(x) = x^3 + x - 1$ satisfies the hypotheses of the Mean Value Theorem on $[0, 2]$. Find a c in this interval that satisfies the conclusions of that theorem.

6. (§4.2) Can the equation $4 + 3x + x^7 = 0$ have *two* distinct roots. (*Hint:* assume there are two distinct roots, and show a contradiction.)

7. (§4.2) Using the Mean Value Theorem, find upper and lower bounds on $f(4)$ for continuous, differentiable function $f(x)$ with the properties that $f(0) = 1$, and $-3 \le f'(x) \le 1$ for x in $[0, 4]$.

8. (§4.2) Let $f(x) = |x-2|$. Show there is no c such that $f(4) - f(1) = f'(c)(4-1)$. Doesn't this contradict the Mean Value Theorem?

9. (§4.10) Find the most general antiderivative of the following:

$$f(x) = x^3 - 2, \qquad g(x) = \sqrt{2x} + x^{-4}, \qquad h(x) = 4\cos x - \sin x,$$
$$j(x) = 1 + x^{3/5}, \qquad k(x) = 4/(1+x^2)$$

10. (§5.2) Estimate $\int_0^1 x^2\,\mathrm{d}x$ by a Riemann sum, using 4 subintervals, and using the right hand endpoint rule. Is this an underestimate or overestimate of the area?

11. (§5.3) State the Fundamental Theorem of Calculus, parts 1 and 2.

12. (§5.3) Find the derivative of $f(x) = \int_1^x \sqrt{1+t^3}\,\mathrm{d}t$.

13. (§5.3) Find the derivative of $f(x) = \int_x^3 \sin\left(\sqrt[4]{t}\right)\,\mathrm{d}t$.

14. (§5.3) Find the derivative of $f(x) = \int_0^{x^2} \sqrt{1+t^4}\,\mathrm{d}t$.

15. (§5.3) Let $f(x) = \int_{1/x}^{1+x^2} t^3\,\mathrm{d}t$. Evaluate $f'(2)$. (*Hint:* split the integral as $f(x) = \int_{1/x}^{1} t^3\,\mathrm{d}t + \int_1^{1+x^2} t^3\,\mathrm{d}t$, and use the Fundamental Theorem twice.)

16. (§5.3) Give a physical interpretation of the definite integral $\int_1^2 x^2\,\mathrm{d}x$. Make a drawing (graph) to show this interpretation.

17. (§5.3) Evaluate the following definite integrals:

$$\int_2^4 \mathrm{d}x \quad \int_0^1 6x^2\,\mathrm{d}x \quad \int_\pi^{2\pi} \cos\theta\,\mathrm{d}\theta \quad \int_0^1 \left(3 + x\sqrt{x}\right)\,\mathrm{d}x \quad \int_1^2 \frac{3+x^2}{x^3}\,\mathrm{d}x \quad \int_1^e t^{-1}\,\mathrm{d}t$$

18. (§5.4) Evaluate the indefinite integrals: (Don't forget the "$+C$")

$$\int x^n\,\mathrm{d}x, \text{ with } n \neq -1, \quad \int x^{-1}\,\mathrm{d}x, \quad \int e^x\,\mathrm{d}x, \quad \int \sin x\,\mathrm{d}x, \quad \int \cos x\,\mathrm{d}x,$$
$$\int 1/(1+x^2)\,\mathrm{d}x, \quad \int \sec^2 x\,\mathrm{d}x, \quad \int \sec x \tan x\,\mathrm{d}x,$$

19. (§5.5) Evaluate the indefinite and definite integrals:

$$\int x^2\left(x^3-1\right)^6\,\mathrm{d}x, \quad \int \frac{x+5x^4}{\sqrt{1+x^2+2x^5}}\,\mathrm{d}x, \quad \int \frac{\sin x}{1+\cos^2 x}\,\mathrm{d}x, \quad \int \frac{1}{x\ln x}\,\mathrm{d}x,$$
$$\int \sqrt{4+3x}\,\mathrm{d}x, \quad \int e^x \sin\left(1+e^x\right)\,\mathrm{d}x, \quad \int \frac{e^{1/x}}{x^2}\,\mathrm{d}x,$$
$$\int_0^2 (x-1)^4\,\mathrm{d}x, \quad \int_0^{\sqrt{\pi}} x\cos x^2\,\mathrm{d}x, \quad \int_0^\pi \sin x \sin\left(\cos x\right)\,\mathrm{d}x, \quad \int_1^{e^\pi} \frac{\cos\left(\ln x\right)}{x}\,\mathrm{d}x,$$

20. (§5.6) Give the definition of $\ln x$.

21. (§5.6) Use the fact that for $t > 1$,

$$\frac{1}{t^2} \leq \frac{1}{t} \leq \frac{1}{\sqrt{t}}$$

to estimate $\ln 2$.

22. (§6.1) Find the area of each of the regions enclosed by the given curves:

1. $y = 4x,\ y = 1/x,\ x = 1,\ x = 4.$
2. $y = x + 2,\ y = 4 - x^2.$
3. $y = 1/x,\ y = 1/x^2,\ x = 3.$
4. $y = x^2,\ x = y^2.$
5. $y = \sin x,\ y = \sin 2x,\ x = 0,\ x = \pi/2.$ (*Hint:* you may wish to use the fact that $\sin 2x = 2\sin x\cos x$.)

23. (§6.2) Let $A(x)$ be the cross sectional area of shape S, which is bounded by the planes $x = a$, $x = b$. What is the volume of S?

24. (§6.2) Find the volume of each of the following shapes:

1. Rotation of region bounded by $y = \sqrt{x}$, $x = 0$, $x = 2$ around the x-axis.
2. Rotation of region bounded by $y = \sqrt{\ln x^{1/x}}$, $x = 1$, $x = e$ around the x-axis.
3. Rotation of region bounded by $y = x^2$, $y = 0$, $y = 4$ around the y-axis.
4. Rotation of region bounded by $y = x^2$, $x = 0$, $x = 2$ around the y-axis.
5. Rotation of region bounded by $y = x^2$, $y = 4$, around the line $y = 4$.
6. Rotation of region bounded by $y = x$, $y = \sqrt{x}$, around the line $x = 2$.

25. (§6.5) Find the average value of the function on the given interval:

1. $f(x) = \sqrt{x}, \quad [0, 2].$
2. $f(x) = x\cos\left(x^2/2\right), \quad [0, \sqrt{\pi}].$

26. (misc.) Give an upper bound on $\int_0^1 \sin\sqrt[4]{t}\,dt$. Give a lower bound on this integral using the fact that $\sin x \geq x/2$ for x in $[0, 1]$.

Exam 2 Prep Sheet:

27. (§7.1) Integration by parts: (a) $\int xe^x\,dx$. (b) $\int x^2e^x\,dx$. (c) $\int x\sin x\,dx$. (d) $\int x\ln x\,dx$. (e) $\int \arctan 4x\ dx$. (f) $\int_0^1 (x^2+1)e^{-x}\,dx$. (g) $\int_1^2 x^{-2}\ln x\,dx$. (h) $\int \sin x\ln(\cos x)\ dx$. (i) $\int \cos(\ln x)\ dx$.

28. (§10.3) Polar Coordinates: (a) Convert the point $(1, 2)$ to polar coordinates. (b) Convert the point $(-2, 0)$ to polar coordinates. (c) Convert the point $(2, \pi/3)$ to Cartesian coordinates. (d) Convert the point $(1.5, 5\pi/4)$ to Cartesian coordinates. (e) Graph the curve given by the polar equation $r = 2$. (f) Graph the curve given by the polar equation $r = \sin\theta$. (g) Graph the curve given by the polar equation $r = \sin 2\theta$.

29. (Appendix G) Arithmetic with Complex numbers. Evaluate the following:

(a) $(4 + 3i) + (2 - 5i)$.
(b) $(1 - 3i)(3 - 5i)$.
(c) $(5 - 2i) - (3 - i)$.
(d) $(-1 + i)\,/\,(5 - i)$
(e) $(12(\cos 8\pi/5 + i\sin 8\pi/5))\,/\,(3(\cos\pi/10 + i\sin\pi/10))$
(f) $3(\cos\pi/3 + i\sin\pi/3)2(\cos\pi/2 + i\sin\pi/2)$.
(g) $[2(\cos 2\pi/3 + i\sin 2\pi/3)]^4$.
(h) $[\cos(-\pi/6) + i\sin(-\pi/6)]^8$.

(i) $[3(\cos\pi/3 + i\sin\pi/3)]^3$.

30. (Appendix G) Complex numbers: modulus, argument, conjugate: (a) Convert $2+i$ to polar form, *i.e.*, find r,θ such that $2+i = r(\cos\theta + i\sin\theta)$. (b) Convert $-1-i$ to polar form. (c) Convert $3(\cos 7\pi/6 + i\sin 7\pi/6)$ to $a+bi$ form. (d) What is $|z|$ when $z = 3+4i$? (e) What is $\overline{z}$ when $z = 2+10i$? (f) What is $\overline{4}$? (g) Give the four complex fourth roots of -2. (h) Give the three complex cube roots of 8. (i) Find the five zeroes to the polynomial x^5+32.

31. (Supplement §1) Euler's Formula:

1. Evaluate $e^{3+2i\pi}$.

32. (§7.2) Questions of the form $\int \cos^m x \sin^n x \, dx$. There are two main cases:

(I) If one of n or m is odd, then keep one power of the appropriate cos or sin, and convert the rest using $\cos^2 x + \sin^2 x = 1$. Then make a u substitution.
For example you should convert $\int \cos^5 x \sin^2 x \, dx$ to $\int \cos x \left(1-\sin^2 x\right)^2 \sin^2 x \, dx$ and use $u = \sin x$.

(II) If both n and m are even, then use the identities $\cos^2\theta = \frac{1+\cos 2\theta}{2}$, $\sin^2\theta = \frac{1-\cos 2\theta}{2}$. to lower the powers.

Questions of this kind: (a) $\int \cos^3 x \sin^1 x \, dx$. (b) $\int \cos^2 x \sin^3 x \, dx$. (c) $\int \sec^6 x \sin^5 x \, dx$. (d) $\int \cos^2 x \sin^4 x \, dx$. (e) $\int_0^{\pi/2} \cos^5 x \, dx$. (f) $\int \sin^3 x \, dx$. (g) $\int \frac{\sin^3(\ln x)}{x} \, dx$. (h) $\int \sin^4 x \, dx$. (i) $\int \cos^4 x \, dx$. (j) $\int \sin^8 x \, dx$.

33. (§7.2) Questions of the form $\int \tan^m x \sec^n x \, dx$. There are three cases:

(I) If m is odd, keep a $\sec x \tan x$, and convert $\tan^{m-1} x$ to powers of $\sec x$ using $\tan^2 x + 1 = \sec^2 x$. Use the u substitution $u = \sec x$.

(II) If n is *even*, keep a $\sec^2 x$, and convert $\sec^{n-2} x$ to powers of $\tan x$ using $\tan^2 x + 1 = \sec^2 x$. Use the u substitution $u = \tan x$.

(III) If m is even and n is odd, protest and complain! This case is difficult. One thing you can do is keep a $\sec x$, and convert $\cos^{1-n} x$ to a denominator in $\sin x$ using $\sin^2 x + \cos^2 x = 1$, use $u = \sin x$, then try a partial fraction expansion. Not very pretty.

Questions of this kind: (a) $\int \tan^3 x \sec x \, dx$. (b) $\int \tan^3 x \sec^3 x \, dx$. (c) $\int \tan^2 x \sec^4 x \, dx$. (d) $\int \tan^3 x \sec^6 x \, dx$. (e) $\int \tan^2 x \, dx$. (f) $\int \sec^3 x \, dx$.

34. (§7.3) 'Hidden' Trig integrals. In these there is usually a component that looks like Pythagora's Theorem, *e.g.*, a $\sqrt{x^2+9}$ or a $\sqrt{25-x^2}$. Draw a triangle with the right sides, make θ one of the angles and express the integral in terms of θ. (a) $\int \sqrt{100-x^2} \, dx$. (b) $\int \frac{dx}{\sqrt{9+x^2}}$. (c) $\int \frac{x\,dx}{\sqrt{9+x^2}}$. (d) $\int \frac{10\,dx}{x(100+x^2)^{3/2}}$. (e) $\int_0^2 x^3\sqrt{x^2+4} \, dx$. (f) $\int \frac{\sqrt{1+x^2}\,dx}{x}$.

35. (§7.3) Sometimes the Hidden Trig integrals are obscured. In these you need to make a substitution to get it to the right form: (a) $\int \frac{x-2}{x^2-2x+5} \, dx$. (Note that the denominator is irreducible, so a Partial Fraction Expansion will do no good here. Instead, try $u = x-2$.) (b) $\int \frac{dx}{(x+3)\sqrt{x^2+6x-25}}$. (c) $\int_{-3}^{5} \frac{dx}{(55-6x-x^2)^{3/2}}$.

36. (§7.4) Partial Fraction Expansion, *i.e.*, questions of the form $\int \frac{p(x)}{q(x)} \, dx$ for polynomials p,q. There is a three step strategy:

(I) If $\deg(p) \geq \deg(q)$, then do a 'long division with remainder' to find polynomials s, r such that $p(x)/q(x) = s(x) + r(x)/q(x)$, and $\deg r < \deg q$. Since s can be integrated easily, we need not look at it anymore.

(II) Express $q(x)$ as linear and irreducible quadratic factors. (In this class, the factoring will be either trivial *e.g.*, $q(x) = x^3 - 9x$, or q will be given to you factored.)

(III) Fraction magic! Remember that a linear factor $ax + b$ corresponds to a summand $\frac{K}{ax+b}$, while an irreducible quadratic factor $ax^2 + bx + c$ corresponds to a summand $\frac{Kx+L}{ax^2+bx+c}$. Repeated factors are repeated in the sum.

Examples:

$$\frac{x^2+3x-2}{(x-1)x(x+4)} \Rightarrow \frac{A}{x-1} + \frac{B}{x} + \frac{C}{x+4}$$
$$\frac{2x^2-3}{(x+1)^3(2x-3)} \Rightarrow \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3} + \frac{D}{2x-3}$$
$$\frac{x^4+3x-2}{(x^2+1)(3x+7)(x^2+3x+4)} \Rightarrow \frac{Ax+B}{x^2+1} + \frac{C}{3x+7} + \frac{Dx+E}{x^2+3x+4}$$
$$\frac{x^5-3x^4-2x}{x(x^2+2)^2(x-2)(x+1)^2} \Rightarrow \frac{A}{x} + \frac{Bx+C}{x^2+2} + \frac{Dx+E}{(x^2+2)^2} + \frac{F}{x-2} + \frac{G}{x+1} + \frac{H}{(x+1)^2}.$$

Questions of this kind: (a) $\int \frac{x^4+2x^3-3}{x(x-1)(x+3)}\,dx$. (b) $\int \frac{x^3-2x^3-3}{(x-2)(x+1)^2(2x+1)}\,dx$. (c) $\int \frac{2x^2-x+1}{(x^2+3)(x-1)}\,dx$. (d) $\int \frac{-x^2+10x-1}{(x^2+8)(x+1)^2}\,dx$. (e) $\int \frac{-x^3-2x}{(x^2+1)^2(x-1)^2}\,dx$. (f) $\int \frac{x^2}{(x+1)^3}\,dx$. (g) $\int_0^1 \frac{x^3-4x-10}{x^2-x-6}\,dx$. (h) $\int_0^1 \frac{x}{x^2+4x+13}\,dx$. (i) $\int \frac{x^2}{x^2+1}\,dx$. (j) $\int \frac{dx}{x^4+x^2}\,dx$. (k) $\int \frac{x^2+3x-2}{(x-1)x(x+4)}\,dx$. (l) $\int \frac{2x^2-3}{(x+1)^3(2x-3)}\,dx$.

37. (§7.4) Partial Fraction Expansion may also be obscured. Consider the example

$$\int \frac{dx}{\sqrt[5]{x} + \sqrt[3]{x}}.$$

In this case if you let $u^{15} = x$, you can rewrite this integral as

$$\int \frac{15u^{14}\,du}{u^3 + u^5}.$$

which is a rational function.
Questions of this kind: (a) $\int \frac{e^{3x}-e^{2x}+6e^x}{e^{3x}-e^x}\,dx$. (b) $\int \frac{\sin x}{1+\cos^2 x}\,dx$.

38. (§7.5) In reality it may not be immediately obvious how to solve an integral, as they will not be given to you with the relevant section numbers. Thus you should look over the strategies given in §7.5 to approach integrals.
(a) $\int \frac{e^{2x}+4e^x}{e^{2x}+2e^x+5}\,dx$. (b) $\int \sqrt{3-2x-x^2}\,dx$. (c) $\int_0^{\pi/4} \tan^5 x \sec^3 x\,dx$. (d) $\int \arctan x\,dx$. (e) $\int \frac{x^4}{x^{10}+16}\,dx$. (f) $\int \frac{1+e^x}{1-e^x}\,dx$. (g) $\int x\sin x\,dx$. (h) $\int (1+\sqrt{x})^3\,dx$. (i) $\int x^2 \sin x\,dx$. (j) $\int x\sin^2 x\,dx$. (k) $\int \frac{x}{\sqrt{1-x^2}}\,dx$. (l) $\int \frac{x}{x^4+4x^2+3}\,dx$. (m) $\int \ln\left|\frac{x^3-x}{x+1}\right|\,dx$. (n) $\int_{-1}^1 x^{-3}\,dx$. (o) $\int (\ln x)^2\,dx$. (p) $\int e^x \cos^3 e^x\,dx$. (q) $\int \frac{dx}{x+\sqrt[3]{x}}\,dx$. (r) $\int \sin\sqrt{x}\,dx$.

39. (§7.7) Approximate Integration. (a) Approximate the definite integral $\int_0^2 \frac{1}{1+x^2}\,dx$. using the Trapezoidal Rule, the Midpoint Rule, and Simpson's Rule, with $n = 4$. (Compare to the real answer $\arctan 2 \approx 1.1071$.) (b) Approximate the definite integral $\int_0^1 e^{x^2}\,\mathrm{d}x$. using the Trapezoidal Rule, the Midpoint Rule, and Simpson's Rule, with $n = 6$.

Exam Finale Prep Sheet:

40. (§7.8) Improper Integrals. Evaluate the following integrals. Tell which are divergent and which are convergent. For convergent integrals, find their value.

$$(a) \int_1^2 \frac{1}{\sqrt{x-1}}\,\mathrm{d}x, \qquad (b) \int_{-1}^3 \frac{1}{x^2}\,\mathrm{d}x, \qquad (c) \int_1^\infty \frac{x}{1+x^2}\,\mathrm{d}x, \qquad (d) \int_{-\infty}^\infty \frac{1}{1+x^2}\,\mathrm{d}x,$$

$$(e) \int_1^\infty \frac{\ln x}{x}\,\mathrm{d}x, \qquad (f) \int_8^9 \frac{1}{\sqrt[3]{x-9}}\,\mathrm{d}x, \qquad (g) \int_0^\pi \sec x\,\mathrm{d}x, \qquad (h) \int_{-1}^1 \frac{e^x}{e^x-1}\,\mathrm{d}x.$$

41. (§8.1) Arc length. Recall that the arc length of the graph of $y = f(x)$ for $a \le x \le b$ is

$$\int_a^b \sqrt{1+(f'(x))^2}\,\mathrm{d}x.$$

(a) Find the arc length of the graph of $y = x^3/3 + 1/(4x)$ for $1 \le x \le 2$. (b) Find the arc length of the graph of $y = \ln(\sec x)$ for $0 \le x \le \pi/3$. (c) Find the arc length of the graph of $x^2 = 4(y+4)^3$ for $0 \le y \le 2$ with $x > 0$. (d) Find the arc length of the graph of $y = x^2/2$ for $1 \le x \le 2$.

42. (§8.2) Surface area of a surface of revolution. Recall that the surface area of the solid obtained by revolving $y = f(x)$ for $a \le x \le b$ about the x-axis is given by

$$2\pi \int_a^b f(x)\sqrt{1+(f'(x))^2}\,\mathrm{d}x.$$

(a) Find the surface area of the surface obtained by revolving $y = \sqrt{r^2 - x^2}$ for $-r \le x \le r$ about the x axis. (b) Find the surface area of the surface obtained by revolving $y = x$ for $0 \le x \le r$ about the x axis. (c) Find the surface area of the surface obtained by revolving $y = x^3/3$ for $0 \le x \le 2$ about the x axis. (d) Find the surface area of the surface obtained by revolving $x = y^4 + 1/(32y^2)$ for $1 \le y \le 2$ about the y axis. (e) Find the surface area of the surface obtained by revolving $y = \sqrt{x}$ for $1 \le x \le 4$ about the x axis.

43. (§8.3) Center of Mass. Recall that the center of mass of the region between the curves $y = f(x)$, $y = 0$, $x = a$, $x = b$ is the point $(\overline{x}, \overline{y})$ where

$$\overline{x} = \frac{\int_a^b x\,f(x)\,\mathrm{d}x}{\int_a^b f(x)\,\mathrm{d}x}, \qquad \overline{y} = \frac{\frac{1}{2}\int_a^b (f(x))^2\,\mathrm{d}x}{\int_a^b f(x)\,\mathrm{d}x}.$$

Find the center of mass of the following regions: (a) the region bounded by $y = 4 - x^2$ and the x axis; (b) the region bounded by $y = e^x$, the x axis, $x = 0$, and $x = 1$;

44. (§8.5) Probability. A random variable, X has probability distribution function f if the probability that X takes value in $[a, b]$ is

$$P(a \leq X \leq b) = \int_a^b f(x)\,\mathrm{d}x.$$

The function $f(x)$ should have these properties: $f(x) \geq 0$ for all x, and

$$\int_{-\infty}^{\infty} f(x)\,\mathrm{d}x = 1.$$

Verify that the following functions are probability density functions:
(a) for any $\alpha < \beta$,

$$f(x) = \begin{cases} \frac{1}{\beta-\alpha} & \text{if } \alpha \leq x \leq \beta, \\ 0 & \text{otherwise.} \end{cases}$$

(b) $f(x) = \frac{1}{\pi}\frac{1}{1+x^2}$.
(c) for any $j > 1$,

$$f(x) = \begin{cases} (j-1)x^{-j} & \text{if } 1 \leq x, \\ 0 & \text{otherwise,} \end{cases}$$

45. (§8.5) Probability. The *mean* (or expected value) of a random variable, X, with probability distribution function f, is

$$\int_{-\infty}^{\infty} x f(x)\,\mathrm{d}x.$$

Find the mean of each of the following random variables:
(a) X with probability distribution function

$$f(x) = \begin{cases} \frac{1}{\beta-\alpha} & \text{if } \alpha \leq x \leq \beta, \\ 0 & \text{otherwise.} \end{cases}$$

(b) X with probability distribution function

$$f(x) = \begin{cases} (j-1)x^{-j} & \text{if } 1 \leq x, \\ 0 & \text{otherwise,} \end{cases}$$

for any $j > 1$.

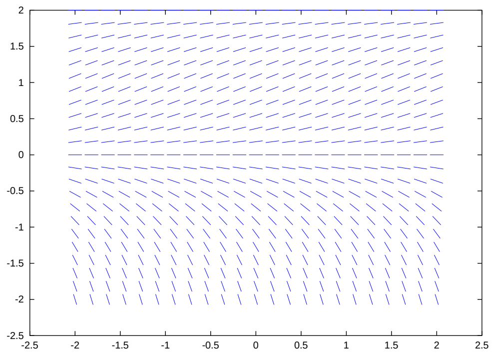
46. (§10.5) Conic Sections. Identify the following equations as being those of an ellipse, parabola, or hyperbola. Then sketch the graph of each of them.
(a) $4x^2 = -y$,

(b) $(x/4)^2 + (y/9)^2 = 1$,
(c) $x = y^2 - 4y + 4$,
(d) $-(x/9)^2 + (y/1)^2 = 1$,
(e) $x^2 + y^2 = 100$,
(f) $-x^2 + y^2 = 36$,

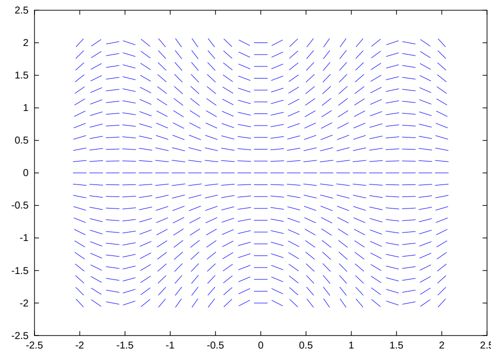
47. (§9.2) Direction Fields. Consider the four direction fields plotted in Figure 1. Match each field to one of the following differential equations:
(I) $y' = y\sin(2x)$,
(II) $y' = y^2 - x^2$,
(III) $y' = y\,(1 - y/2)$,
(IV) $y' = 1 - xy$.

48. (§9.2) Direction Fields. On each of the direction fields plotted in Figure 1, sketch the graph of the function satisfying the differential equation and which goes through the point $(0, 0.5)$.

49. (§9.3) Separable Differential Equations.
Solve the following differential equations by separation.
(a) $y' = yx$,
(b) $y' = x^2/y^2$,
(c) $y' = y^2 \sin x$,
(d) $y' = \sqrt{xy}$,



(a)



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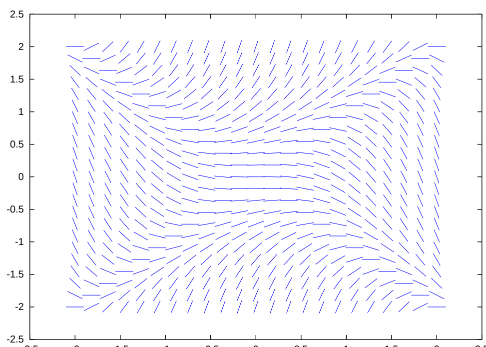

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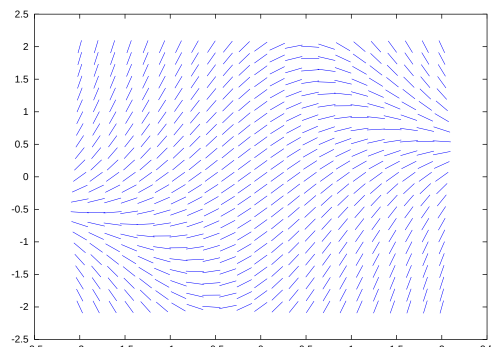

(d)

Figure 1: Four direction fields.