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W 2004 M20C : Calculus and Analytic Geometry

Steven Pav

Exam 1

SPAV@MATH.UCSB.EDU

Name \_\_\_\_\_ Student # \_\_\_\_\_ Section \_\_\_\_\_

Instructions: Read all instructions carefully. Write your name, student number, and section above. Clearly indicate all your answers, and show all your work; for many problems partial credit is available for partially correct answers. 9 Problems in all. Total Points: 90.

You may find the following information helpful: For the vectors  $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ ,  $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$ , the following relations hold:

$$\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3 \quad \mathbf{a} \times \mathbf{b} = \langle a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1 \rangle$$

$$|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}.$$

**Multiple Choice. Clearly circle your answers here. No partial credit.**

P1 (5 pts) Which of the following expressions is another form for  $\mathbf{a} \cdot \mathbf{b}$ ?

- (a)  $|\mathbf{a}||\mathbf{b}|$       (b)  $|\mathbf{a}||\mathbf{b}|\cos\theta$       (c)  $|\mathbf{a}||\mathbf{b}|\sin\theta$       (d)  $|\mathbf{a}||\mathbf{b}|\tan\theta$

P2 (5 pts) Which of the following is a vector parallel to  $\mathbf{a} = \langle 1, -2, 5 \rangle$ ?

- (a)  $\langle \alpha, \alpha, 3 \rangle$       (b)  $\langle 0, 2, -5 \rangle$       (c)  $\langle -2, 4, -10 \rangle$       (d)  $\mathbf{a} \times \langle -2, 5, 1 \rangle$

P3 (5 pts) Which of the following expresses a vector perpendicular to both  $\mathbf{a}$  and  $\mathbf{b}$ ?

- (a)  $\mathbf{a} \times \mathbf{b}$       (b)  $\mathbf{a} + \mathbf{b}$       (c)  $\mathbf{a} \cdot \mathbf{b}$       (d)  $|\mathbf{a}|\mathbf{b} + |\mathbf{b}|\mathbf{a}$

**Essay Questions. Show all your work. Partial credit is available.**

P4 (10 pts) Find a unit length vector in the same direction as  $\mathbf{v} = \langle -3, 12, 4 \rangle$ .

P5 (10 pts) Find the point in the plane  $2x + 3y - z = 14$  that is closest to the origin.

P6 (10 pts) Let  $\theta$  be the angle subtended by the two planes given by  $-x + 2y - z = 12$  and  $3x + z = 1$ . Find  $\theta$ . (Your answer may include a single arcsin, arccos, or arctan.) (Note there are actually *two* answers,  $\theta_1, \theta_2$ , with  $\theta_1 + \theta_2 = \pi$ . You may find either.)

P7 (10 pts) Find a vector perpendicular to the two lines parametrized by

$$\mathbf{r}_1(t) = \langle 0, 1, 3 \rangle + t \langle -1, 0, 1 \rangle \quad \text{and} \quad \mathbf{r}_2(t) = \langle 0, 1, 3 \rangle + t \langle 2, 1, -1 \rangle.$$

P8 (10 pts) Find the limit

$$\lim_{t \rightarrow 0} \langle 5t, \cos t, |t - 1| \rangle$$

P9 (25 pts) Let  $\mathbf{r}(t) = \langle \sqrt{3}t^2, 2 + t^3, 2t \rangle$ .

- Find  $\mathbf{r}'(t)$ .
- Find  $\mathbf{T}(1)$ .
- Find the arc length of the curve  $C$  parametrized by  $\mathbf{r}(t)$  for  $0 \leq t \leq 10$ .