2004/01/26	**W 2004 M20C : Calculus and Analytic Geometry**	
Steven Pav	**Exam 1**	SPAV@MATH.UCSD.EDU

Name _____ Student #_____ Section _____

Instructions: Read all instructions carefully. Write your name, student number, and section above. Clearly indicate all your answers, and show all your work; for many problems partial credit is available for partially correct answers. 9 Problems in all. Total Points: 90.

You may find the following information helpful: For the vectors $\boldsymbol{a} = \langle a_1, a_2, a_3 \rangle$, $\boldsymbol{b} = \langle b_1, b_2, b_3 \rangle$, the following relations hold:

$$\boldsymbol{a} \cdot \boldsymbol{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 \qquad \boldsymbol{a} \times \boldsymbol{b} = \langle a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1 \rangle$$
$$|\boldsymbol{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}.$$

Multiple Choice. Clearly circle your answers here. No partial credit.

P1 (**5 pnts**) Which of the following expressions is another form for $|\boldsymbol{a} \times \boldsymbol{b}|$?
(a) $|\boldsymbol{a}|\,|\boldsymbol{b}|$ (b) $|\boldsymbol{a}|\,|\boldsymbol{b}| \tan\theta$ (c) $|\boldsymbol{a}|\,|\boldsymbol{b}| \sin\theta$ (d) $|\boldsymbol{a}|\,|\boldsymbol{b}| \cos\theta$

P2 (**5 pnts**) Which of the following is a vector perpendicular to $\boldsymbol{a} = \langle 1, -2, 5 \rangle$?
(a) $\langle 1, 1, 1 \rangle$ (b) $\langle 6, 3, 0 \rangle$ (c) $\langle -2, 4, -10 \rangle$ (d) $\boldsymbol{a} \cdot \langle -2, 5, 1 \rangle$

P3 (**5 pnts**) Which of the following expresses a plane perpendicular to $\boldsymbol{n}$?
(a) $t\,(\boldsymbol{n} \times \langle x, y, z \rangle)$ (b) $\boldsymbol{n} - \langle x, y, z \rangle$ (c) $\boldsymbol{n} \cdot \langle x, y, z \rangle = 12$ (d) $\langle x, y, z \rangle = \boldsymbol{n}$

Essay Questions. Show all your work. Partial credit is available.

P4 (**10 pnts**) Find the point in the plane $1x - 2y + 5z = 15$ that is closest to the origin.

P5 (**10 pnts**) Find a unit length vector in the same direction as $\boldsymbol{v} = \langle -4, -2, 4 \rangle$.

P6 (**10 pnts**) Let θ be the angle subtended, at their point of intersection, by the two lines

$$\boldsymbol{r_1}(t) = \langle 0, 1, 3 \rangle + t \langle -1, 0, 1 \rangle \quad \text{and} \quad \boldsymbol{r_2}(t) = \langle 0, 1, 3 \rangle + t \langle 2, 1, -1 \rangle .$$

Find θ. (Your answer may include a single arcsin, arccos, or arctan.) (Note there are actually *two* answers, θ_1, θ_2, with $\theta_1 + \theta_2 = \pi$. You may find either.)

P7 (**10 pnts**) Find the equation of the plane through the points $P\,(0,0,0)$, $Q\,(1,1,3)$, $R\,(-1,0,1)$.

P8 (**10 pnts**) Find the limit

$$\lim_{t \to \infty} \left\langle \frac{5}{t}, e^{-t}, \frac{3t-1}{2t} \right\rangle$$

P9 (**25 pnts**) Let $\boldsymbol{r}(t) = \left\langle 2t^3, 4 + \sqrt{6}\, t^2, 2t \right\rangle$.

- Find $\boldsymbol{r}'(t)$.
- Find $\boldsymbol{T}(1)$.
- Find the arc length of the curve C parametrized by $\boldsymbol{r}(t)$ for $0 \leq t \leq 2$.