2004/02/23	**W 2004 M20C : Calculus and Analytic Geometry**	
Steven Pav	**Exam 2**	SPAV@MATH.UCSD.EDU

Instructions: Read all instructions carefully. Write your name, student number, and section number on your answer sheet. Clearly indicate your answers & show all your work; for many problems partial credit is available. 8 Problems worth 110 Points.

$$s = \int \left|\boldsymbol{R}'(t)\right| \mathrm{d}t \qquad \nabla f = \langle f_x, f_y \rangle \qquad \mathrm{D}_{\boldsymbol{u}} f = \nabla f \cdot \boldsymbol{u} \qquad D = f_{xx} f_{yy} - f_{yx} f_{xy}$$

Multiple Choice; Write answer on your answer sheets; No partial credit.

P1 (**5 pnts**) Which of the following points is in the domain of $f(x, y) = \frac{\log(xy)}{\sqrt{x^2 - y^2}}$?

(a) $(3, 3)$ (b) $(-3, 2)$ (c) $(3, 2)$ (d) $(2, 3)$.

P2 (**5 pnts**) Which of the following limits does *NOT* exist?

(a) $\lim_{(x,y)\to(0,0)} \frac{x^3y^2 - x^2y}{1 + x^2 + y^2}$ (b) $\lim_{(x,y)\to(0,0)} \frac{x^4 - y^4}{x^2 + y^2}$ (c) $\lim_{(x,y)\to(0,0)} \frac{x^2 - y^2}{x^2 + y^2}$.

Problems. Show all work on your answer sheets. Partial credit is available.

P3 (**10 pnts**) Let $z = e^x \sin y$ with $x = s^2 t, y = st^2$. Find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$.

P4 (**10 pnts**) Find the arc length of the curve C parametrized by $\boldsymbol{R}(t) = \langle 3\cos t, 3\sin t, t \rangle$ for $0 \le t \le 4\pi$.

P5 (**20 pnts**) Let $f(x, y) = \sqrt{19 - x^2 - y^2}$

- Find the linearization, $L(x, y)$ of $f(x, y)$ at $(1, 3)$.
- Use this linearization to approximate $\sqrt{19 - (0.9)^2 - (3.1)^2}$.

P6 (**20 pnts**) Let $P(x, y) = x^2y^2 - xy + 1$ be the pressure at point (x, y) in a pool, measured in (a unit called) pascals. Let position be measured in meters.

- Find the gradient of P, *i.e.*, find $\nabla P(x, y)$.
- A fish is located at $(1, 2)$, and wants to move in the direction of maximum *decrease* in pressure. Which direction is that? Express your answer as a unit vector.
- What is the maximum instantaneous rate of decrease in pressure at $(1, 2)$, measured in pascals per meter?

P7 (**20 pnts**) Let $f(x, y) = xy(1 - x - y)$.

- Find all the critical points of $f(x, y)$.
- Use the second derivative test to classify each of these as a local maximum, local minimum, or neither.

P8 (**20 pnts**) Let $f(x, y) = xy(1 - x - y)$. Suppose we want to find the maximum and minimum values of f on the circle $x^2 + y^2 = \frac{1}{3}$. Consider the method of Lagrange Multipliers:

- Set up two equations involving x, y, λ that should be solved to find the extremal values.
- Find two points, say (x_1, y_1) and (x_2, y_2), that satisfy these equations. *Hint:* Exploit the symmetry of the problem.
- Evaluate $f(x, y)$ at these points, and make a conjecture about the extremal values of f on this circle.
- Find more points that satisfy the Lagrange equations *OR* show that these two points are the only solutions of the Lagrange equations.