2004/02/23	**W 2004 M20C : Calculus and Analytic Geometry**	
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Instructions: Read all instructions carefully. Write your name, student number, and section number on your answer sheet. Clearly indicate your answers & show all your work; for many problems partial credit is available. 8 Problems worth 110 Points.

$$s = \int \left|\boldsymbol{R}'(t)\right| \mathrm{d}t \qquad \nabla f = \langle f_x, f_y \rangle \qquad \mathrm{D}_{\boldsymbol{u}} f = \nabla f \cdot \boldsymbol{u} \qquad D = f_{xx} f_{yy} - f_{yx} f_{xy}$$

Multiple Choice; Write answer on your answer sheets; No partial credit.

P1 (**5 pnts**) Which of the following points is in the domain of $f(x,y) = \frac{\log(x+y)}{\sqrt{y^2+x^2-4}}$?

(a) $(-2,-2)$ (b) $(2,0)$ (c) $(2,2)$ (d) $(1,0)$.

P2 (**5 pnts**) Which of the following limits does *NOT* exist?

(a) $\lim_{(x,y)\to(0,0)} \frac{xy^4 - x^4 y}{2+x+y}$ (b) $\lim_{(x,y)\to(0,0)} \frac{x-y}{x+y}$ (c) $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x+y}$.

Problems. Show all work on your answer sheets. Partial credit is available.

P3 (**10 pnts**) Let $z = (\cos x)\, e^y$ with $x = s^3 t, y = st$. Find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$.

P4 (**10 pnts**) Find the arc length of the curve C parametrized by $\boldsymbol{R}(t) = \langle 2\cos t, 2\sin t, t\rangle$ for $0 \le t \le 2\pi$.

P5 (**20 pnts**) Let $f(x,y) = \sqrt{21 - x^2 - y^2}$

- Find the linearization, $L(x,y)$ of $f(x,y)$ at $(1,2)$.
- Use this linearization to approximate $\sqrt{21 - (1.1)^2 - (1.9)^2}$.

P6 (**20 pnts**) Let $P(x,y) = x^3 y + x y^3 + 1$ be the temperature at point (x,y) in a pool, measured in (a unit called) degrees kelvin. Let position be measured in meters.

- Find the gradient of P, *i.e.*, find $\nabla P(x,y)$.
- A fish is located at $(2,1)$, and wants to move in the direction of maximum *decrease* in temperature. Which direction is that? Express your answer as a unit vector.
- What is the maximum instantaneous rate of decrease in temperature at $(2,1)$, measured in degrees kelvin per meter?

P7 (**20 pnts**) Let $f(x,y) = xy\,(1 - x - y)$.

- Find all the critical points of $f(x,y)$.
- Use the second derivative test to classify each of these as a local maximum, local minimum, or neither.

P8 (**20 pnts**) Let $f(x,y) = xy\,(1 - x - y)$. Suppose we want to find the maximum and minimum values of f on the circle $x^2 + y^2 = \frac{1}{3}$. Consider the method of Lagrange Multipliers:

- Set up two equations involving x, y, λ that should be solved to find the extremal values.
- Find two points, say (x_1, y_1) and (x_2, y_2), that satisfy these equations. *Hint:* Exploit the symmetry of the problem.
- Evaluate $f(x,y)$ at these points, and make a conjecture about the extremal values of f on this circle.
- Find more points that satisfy the Lagrange equations *OR* show that these two points are the only solutions of the Lagrange equations.