2004/03/16	**W 2004 M20C : Calculus and Analytic Geometry**	
Steven Pav	**Exam Finale**	SPAV@MATH.UCSD.EDU

Instructions: Read all instructions carefully. Write your name, student number, and section number **on your answer sheet**. You will *not* hand in this sheet: clearly indicate your answers & show all your work on your answer sheet. For many problems partial credit is available. 14 Problems worth 160 Points.

Grading Notes: Please do *not* use your calculator to evaluate integrals; you must show your work to receive full credit. For those questions with multiple parts, please circle or box your answers so the grader does not have to hunt them down.

$$\boldsymbol{a}\cdot\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3 \qquad \boldsymbol{a}\times\boldsymbol{b} = \langle a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1\rangle$$

$$|\boldsymbol{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2} \qquad L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

$$\mathrm{d}A = \mathrm{d}y\mathrm{d}x = r\mathrm{d}r\mathrm{d}\theta \quad s = \int \left|\boldsymbol{R}'(t)\right| \mathrm{d}t \qquad \mathrm{D}_{\boldsymbol{u}}f = \nabla f\cdot\boldsymbol{u} \qquad \nabla f = \langle f_x, f_y\rangle$$

$$D = f_{xx}f_{yy} - f_{yx}f_{xy} \qquad \nabla f = \lambda\nabla g.$$

Multiple Choice: Indicate answers on your answer sheets. No partial credit.

P1 (**5 pnts**) Which of the following integrals has value 0?
(a) $\int_{-1}^{1}\int_{-1}^{1}\left(x^2+y^2+1\right)\mathrm{d}y\mathrm{d}x$, (b) $\int_1^3\int_{-2}^{2}\left(y^2x^3\right)\mathrm{d}y\mathrm{d}x$,
(c) $\int_1^3\int_{-2}^{2}\left(y^2x^3\right)\mathrm{d}x\mathrm{d}y$, (d) $\int_0^2\int_1^2\left(e^x+e^y\right)\mathrm{d}y\mathrm{d}x$.

P2 (**5 pnts**) Which of the following is a line parallel to the vector $\boldsymbol{v} = \langle 1,2,-3\rangle$?
(a) $\langle 1,2,-3\rangle + t\langle 1,1,1\rangle$, (b) $1x+2y-3z=4$,
(c) $\langle 1,1,0\rangle + t\langle -2,-4,6\rangle$, (d) $\langle 0,1,1\rangle + t\langle -2,-2,-2\rangle$.

P3 (**5 pnts**) Which of the following is perpendicular to both $\boldsymbol{a} = \langle 1,1,0\rangle$ and $\boldsymbol{b} = \langle 0,-1,-1\rangle$?
(a) $\boldsymbol{a}\cdot\boldsymbol{b}$, (b) $|\boldsymbol{a}|\,\boldsymbol{b} + |\boldsymbol{b}|\,\boldsymbol{a}$, (c) $\langle 1,1,1\rangle$, (d) $\langle 1,-1,1\rangle$.

P4 (**5 pnts**) Which of the following is a unit vector in the same direction as $\boldsymbol{a} = \langle 2,-4,4\rangle$?
(a) $\left\langle \frac{1}{3}, \frac{-2}{3}, \frac{2}{3}\right\rangle$, (b) $\langle -1,2,-2\rangle$, (c) $\left\langle \frac{1}{2}, -1, 1\right\rangle$, (d) $\left\langle \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, 0\right\rangle$.

P5 (**5 pnts**) Which of the following is the polar point $r = 2, \theta = \frac{\pi}{3}$ expressed in rectangular coordinates?
(a) $(1.999, 0.037)$, (b) $\left(\sqrt{3}, \sqrt{3}\right)$, (c) $\left(\sqrt{3}, 1\right)$, (d) $\left(1, \sqrt{3}\right)$.

Problems. Show all work on your answer sheets. Partial credit is available.

P6 (**10 pnts**) Let $f(x,y) = x + 2y + \cos(xy)$. Find ∇f.

P7 (**10 pnts**) Let $z = e^{(x+y)/2}$ with $x = (s+t)^2, y = (s-t)^2$. Find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$. Put your answer entirely in terms of s and t.

P8 (**10 pnts**) Change the order of the following integral, to make it "x first, then y.":

$$\int_0^3\int_{x^2}^{3x} f(x,y)\mathrm{d}y\mathrm{d}x$$

Exam continues on reverse of page.

P9 (**15 pnts**) Evaluate the triple integral:

$$\int_0^1 \int_x^{2x} \int_0^y 2xyz \, \mathrm{d}z \mathrm{d}y \mathrm{d}x$$

P10 (**15 pnts**) Let $\boldsymbol{R}(t) = \langle 2\cos(t), 2\sin(t), 3t \rangle$ denote the position of a particle at time t.

(a) Find the velocity of the particle.

(b) Find the speed of the particle.

(c) Find the arc length of the curve C parametrized by $\boldsymbol{R}(t)$ for $0 \leq t \leq 1$.

P11 (**15 pnts**) Change the following integral to an integral in polar coordinates.

$$\int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} x + x^2 + y^2 \, \mathrm{d}y \mathrm{d}x$$

Do *not* attempt to solve the integral. Your answer should be entirely in terms of r, θ.

P12 (**15 pnts**) Let $\boldsymbol{R}(t) = \left\langle t^2, t, 2t \right\rangle$ parametrize a curve C. Let P be a plane parametrized by $x + y - 3z = -6$. Find all points of intersection of the curve and plane.

P13 (**20 pnts**) Find the volume of the solid under the surface $z = 2x + y^2$ and above the region in the xy plane bounded by $x = y^2$ and $x = y^3$. *Hint:* your answer should be positive.

P14 (**25 pnts**) You are asked to find the extremal values of $f(x, y) = e^{xy}$ on the set $x^2 + 4y^2 \leq 1$.

(a) Find the gradient ∇f.

(b) Find all the critical points of $f(x, y)$ in the set $x^2 + 4y^2 < 1$.

(c) Use the second derivative test to classify each of these as a local maximum, local minimum, or neither.

(d) Evaluate $f(x, y)$ at the critical points.

Now use the method of Lagrange Multipliers:

(e) Set up equations involving x, y, λ that should be solved to find the extremal values of f on the set $x^2 + 4y^2 = 1$.

(f) Find four points, (x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4), that satisfy these equations.

(g) Evaluate $f(x, y)$ at these points.

(h) Use this information to find the extremal values of $f(x, y)$ on $x^2 + 4y^2 \leq 1$. *Hint:* you may use the fact that if $a < b$ then $e^a \leq e^b$ for all a, b.