Exam 1 Preparation

The following formula will be provided on your exam:

$$\nabla \quad \text{abbreviates} \quad \boldsymbol{i}\frac{\partial}{\partial x} + \boldsymbol{j}\frac{\partial}{\partial y} + \boldsymbol{k}\frac{\partial}{\partial z}$$

Everything else must be committed to memory.

Looking over your homework assignments, I have seen questions of the following types; You should be prepared to answer at least all these questions:

1. What's the difference between a scalar and a vector?
2. Which operations yield scalars, and which vectors?
3. Given $\boldsymbol{v}$ find $|\boldsymbol{v}|$. Find a unit length vector in the same direction as $\boldsymbol{v}$.
4. Given $\boldsymbol{a}$, $\boldsymbol{b}$ find $\boldsymbol{a} \cdot \boldsymbol{b}$. Find $\boldsymbol{a} \times \boldsymbol{b}$. Find $|\boldsymbol{a} \times \boldsymbol{b}|$.
5. Given $\boldsymbol{a}$, $\boldsymbol{b}$ find the angle subtended by the two vectors.
6. Given $\boldsymbol{a}$, $\boldsymbol{b}$ find a unit vector orthogonal to $\boldsymbol{a}$ and $\boldsymbol{b}$.
7. Given point P, and $\boldsymbol{n}$, find the equation of the plane containing P and normal to $\boldsymbol{n}$.
8. Given points P, Q, R, find the equation of the plane containing all three points.
9. Given points P, Q, R, find the area of the triangle ΔPQR.
10. Given vector valued function, $\boldsymbol{R}(t)$, find $\boldsymbol{R}'(t)$ and $\boldsymbol{T}(t)$.
11. Find the arc length of a curve parametrized by $\boldsymbol{R}(t)$ for $a \leq t \leq b$. "Reparametrize the curve in terms of arc length" *i.e.*, find a parametrization, $\boldsymbol{Q}(t)$ of the curve such that the arc length of the curve parametrized by $\boldsymbol{Q}(t)$ for $a \leq t \leq b$, is exactly equal to $b - a$.
12. Given vector valued functions, $\boldsymbol{R}(t), \boldsymbol{Q}(t)$ find all points of intersection of the curves parametrized by the functions. At each point of intersection, find the angles subtended by the curves.
13. What is an isotimic surface of a scalar field? What is a flow line of a vector field?
14. Given scalar field $\phi(x, y, z)$, find the gradient, $\nabla\phi(x, y, z)$. Is $\nabla\phi$ a scalar field or vector field?
15. Given scalar field $\phi(x, y, z)$, and unit vector $\boldsymbol{u}$, find the directional derivative of ϕ in the direction of $\boldsymbol{u}$, *i.e.*, find $\mathrm{D}_{\boldsymbol{u}}\,\phi$.
16. At a point in a scalar field, what is the direction of maximal increase of the scalar field? What is the maximum instantaneous rate of increase of the scalar field, per unit of distance travelled?
17. What is $\nabla[\phi + \psi]$? What is $\nabla[\phi\psi]$? What is $\nabla\left[\phi^k\right]$?
18. Given vector field $\boldsymbol{F}(x, y, z)$, is $\nabla\boldsymbol{F}(x, y, z)$ defined?
19. Given vector field $\boldsymbol{F}(x, y, z)$, find the divergence, $\nabla \cdot \boldsymbol{F}(x, y, z)$. Is $\nabla \cdot \boldsymbol{F}$ a scalar field or vector field? What is $\nabla \cdot [\boldsymbol{F} + \boldsymbol{G}]$? What is $\nabla \cdot [\phi\boldsymbol{F}]$?
20. Given vector field $\boldsymbol{F}(x, y, z)$, find the curl, $\nabla \times \boldsymbol{F}(x, y, z)$. Is $\nabla \times \boldsymbol{F}$ a scalar field or vector field? What is $\nabla \times [\boldsymbol{F} + \boldsymbol{G}]$? What is $\nabla \times [\phi\boldsymbol{F}]$?
21. What is $\nabla \cdot [\nabla\phi]$? What is $\nabla \times [\nabla\phi]$? What is $\nabla \cdot [\nabla \times \boldsymbol{F}]$?
22. Let $\boldsymbol{R}(x, y, z) = \langle x, y, z \rangle$. Let $r(x, y, z) = |\boldsymbol{R}(x, y, z)| = \sqrt{x^2 + y^2 + z^2}$. What is ∇r? What is $\nabla \cdot \boldsymbol{R}$? What are the isotimic surfaces of r? What are the flow lines of $\boldsymbol{R}$?