

Name _____ Student # _____ Section _____

Instructions: Read all instructions carefully. Write your name, student number, and section above. Clearly indicate all your answers, and show all your work; for many problems partial credit is available for partially correct answers. 5 Problems in all. Total Points: 90.

You may find the following information helpful:

$$\nabla \text{ abbreviates } \mathbf{i} \frac{\partial}{\partial x} + \mathbf{j} \frac{\partial}{\partial y} + \mathbf{k} \frac{\partial}{\partial z}$$

Multiple Choice. Clearly circle your answer here. No partial credit.

P1 (5 pnts) Which of the following is a scalar field?

- (a) Gradient of f (b) Divergence of \mathbf{F} (c) Curl of \mathbf{F} .

Essay Questions. Show all your work. Partial credit is available.

P2 (15 pnts) Let C_1 be a regular curve parametrized by $\mathbf{R}_1(t) = \langle t, (3-t)^2, t^3/9 \rangle$. Let C_2

be a regular curve parametrized by $\mathbf{R}_2(t) = \langle 3 \cos(t), -t^4, t+3 \rangle$.

- Find all points of intersection of the curves C_1, C_2 .
- At each point of intersection, find the angle, θ , that the curves subtend. (Your answer may include a single arcsin, arccos, or arctan.) (Note there are actually *two* answers, θ_1, θ_2 , with $\theta_1 + \theta_2 = \pi$. You may find either.)

P3 (15 pnts) Let $f(x, y, z) = (1 + \sin x + y^2) e^{-z}$, represent the concentration of a gas (in grams/cc) at position (x, y, z) , with position measured in centimeters. A creature which breathes the gas is located at $(2\pi, 1, 1)$.

- In what direction should the creature move to most rapidly maximize its environmental gas concentration?
- If the creature begins to move in this direction at a rate of 1/3 cm per second, at what rate will the environmental gas concentration increase?

P4 (25 pnts) • Let $\phi(x, y, z) = \frac{1}{\sqrt{x^2+y^2}}$. Find $\nabla\phi$.

• Let $\mathbf{R}(x, y, z) = y\mathbf{i} - x\mathbf{j}$. Find $\text{div } \mathbf{R}$ and find $\text{curl } \mathbf{R}$.

• Let $\mathbf{Q}(x, y, z) = \frac{1}{\sqrt{x^2+y^2}} \mathbf{R}(x, y, z)$. Find $\text{div } \mathbf{Q}$ and find $\text{curl } \mathbf{Q}$.

P5 (30 pnts) Let the curve C be parametrized by $\mathbf{r}(t) = \langle \cos 2t, \sqrt{5}t, \sin 2t \rangle$. Let $s(\tau)$ be the arc length of the portion of C parametrized by \mathbf{r} for $0 \leq t \leq \tau$.

- Find $\mathbf{r}'(t)$.
- Find $\frac{ds}{d\tau}$.
- Find $\mathbf{T}(t)$.
- Find $s(\tau)$.
- "Reparametrize C in terms of arc length." That is, find some parametrization of C , call it $\mathbf{q}(t)$ such that the arc length of the curve parametrized by $\mathbf{q}(t)$ for $0 \leq t \leq \tau$ is exactly equal to τ .