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| 2004/01/26 | | **W 2004 M20E : Vector Calculus** |
| Steven Pav | **Exam 1** | SPAV@MATH.UCSD.EDU |

Name ______________________ Student #____________ Section ________

Instructions: Read all instructions carefully. Write your name, student number, and section above. Clearly indicate all your answers, and show all your work; for many problems partial credit is available for partially correct answers. 5 Problems in all. Total Points: 90.

You may find the following information helpful:

$$\nabla \quad \text{abbreviates} \quad \boldsymbol{i}\frac{\partial}{\partial x} + \boldsymbol{j}\frac{\partial}{\partial y} + \boldsymbol{k}\frac{\partial}{\partial z}$$

**Multiple Choice. Clearly circle your answer here. No partial credit.**

P1 (**5 pnts**) Which of the following is a vector field?
(a) Divergence of $\boldsymbol{F}$ (b) Gradient of $f$ (c) Laplacian of $f$.

**Essay Questions. Show all your work. Partial credit is available.**

P2 (**15 pnts**) Let $C_1$ be a regular curve parametrized by $\boldsymbol{R_1}(t) = \left\langle t^2, 3+t, 3\cos t\right\rangle$. Let $C_2$ be a regular curve parametrized by $\boldsymbol{R_2}(t) = \left\langle -(2-t)^2, t^2-1, t+1\right\rangle$.

- Find all points of intersection of the curves $C_1, C_2$.
- At each point of intersection, find the angle, $\theta$, that the curves subtend. (Your answer may include a single arcsin, arccos, or arctan.) (Note there are actually *two* answers, $\theta_1, \theta_2$, with $\theta_1 + \theta_2 = \pi$. You may find either.)

P3 (**15 pnts**) Let $f(x,y,z) = \left(4 + \sqrt{2}\sin y + z^3\right)e^{-x}$, represent the concentration of a gas (in grams/cc) at position $(x,y,z)$, with position measured in centimeters. A creature which breathes the gas is located at $(2,\pi,1)$.

- In what direction should the creature move to most rapidly maximize its environmental gas concentration?
- If the creature begins to move in this direction at a rate of 1/6 cm per second, at what rate will the environmental gas concentration increase?

P4 (**25 pnts**)

- Let $\phi(x,y,z) = \frac{1}{\sqrt{y^2+z^2}}$. Find $\nabla\phi$.
- Let $\boldsymbol{Q}(x,y,z) = z\boldsymbol{j} - y\boldsymbol{k}$. Find $\operatorname{div}\boldsymbol{Q}$ and find $\mathbf{curl}\,\boldsymbol{Q}$.
- Let $\boldsymbol{R}(x,y,z) = \frac{1}{\sqrt{y^2+z^2}}\boldsymbol{Q}(x,y,z)$. Find $\operatorname{div}\boldsymbol{R}$ and find $\mathbf{curl}\,\boldsymbol{R}$.

P5 (**30 pnts**) Let the curve $C$ be parametrized by $\boldsymbol{r}(t) = \left\langle \sqrt{7}\,t, -\cos 3t, \sin 3t\right\rangle$. Let $s(\tau)$ be the arc length of the portion of $C$ parametrized by $\boldsymbol{r}$ for $0 \le t \le \tau$.

- Find $\boldsymbol{r}'(t)$.
- Find $\frac{ds}{d\tau}$.
- Find $\boldsymbol{T}(t)$.
- Find $s(\tau)$.
- "Reparametrize $C$ in terms of arc length." That is, find some parametrization of $C$, call it $\boldsymbol{q}(t)$ such that the arc length of the curve parametrized by $\boldsymbol{q}(t)$ for $0 \le t \le \tau$ is exactly equal to $\tau$.