2004/02/23		**W 2004 M20E : Vector Calculus**
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Instructions: Read all instructions carefully. Write your name, student number, and section number on your answer sheet. Clearly indicate your answers & show all your work; for many problems partial credit is available. 7 Problems worth 110 Points.

$$\mathrm{d}\boldsymbol{R} = \boldsymbol{R}'(t)\mathrm{d}t \quad \mathrm{d}\boldsymbol{S} = \left(\frac{\mathrm{d}\boldsymbol{R}}{\mathrm{d}x} \times \frac{\mathrm{d}\boldsymbol{R}}{\mathrm{d}y}\right)\mathrm{d}x\mathrm{d}y \quad \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix}$$

Multiple Choice; Write answer on your answer sheets; No partial credit.

P1 (**5 pnts**) Which of the following is a conservative vector field?
(a) $\boldsymbol{F} = \langle -y, x, 0\rangle$ (b) $\boldsymbol{F} = \langle 2y, 2x, x^2\rangle$ (c) $\boldsymbol{F} = \langle y+z, x+2y, x\rangle$.

Problems. Show all work on your answer sheets. Partial credit is available.

P2 (**10 pnts**) Clearly state the Divergence Theorem *or* Stokes' Theorem. You may write an equation for either theorem, but please state *what the equation means.*

P3 (**20 pnts**) Let C be the curve extending from $(1,0,-1)$ to $(-1,1,0)$, and parametrized by $\boldsymbol{R}(t) = \left\langle \cos(\pi t), \sin\left(\frac{\pi}{2}t\right), t^2-1\right\rangle$. Evaluate

$$\int_C \boldsymbol{F}\cdot \mathrm{d}\boldsymbol{R},$$

where $\boldsymbol{F}(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}\langle x,y,z\rangle$.

P4 (**20 pnts**) Let S be the part of the cone $z^2 = x^2+y^2$ with $1 \le z \le 2$. Let the surface be oriented such that $\boldsymbol{n}\cdot\boldsymbol{k} < 0$, where $\boldsymbol{n}$ is the surface unit normal. Let $\boldsymbol{F} = \left\langle x, y, \frac{z}{2}\right\rangle$. Evaluate

$$\iint_S \boldsymbol{F}\cdot \mathrm{d}\boldsymbol{S}$$

P5 (**20 pnts**) Let R be the region bounded by the y axis, $y = x^2-1$, and $y = 4 - \frac{1}{4}x^2$. Using the transformation $x = uv, y = u^2 - v^2$, rewrite the integral

$$\iint_R e^{x^2+y}\mathrm{d}y\mathrm{d}x$$

as an iterated integral in terms of u, v. Do *not* attempt to evaluate the integral. For full credit *you must write the limits of integration.* You may assume the transformed region $\tilde{R}$ is in the first quadrant of u-v space.

P6 (**25 pnts**) Let R be the region bounded by the inequalities: $\frac{1}{4}x^2 + \frac{1}{9}y^2 + z^2 \le 16$, and $z \ge 1$. Find the volume of R *i.e.*, find

$$V = \iiint_R \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$$

P7 (**10 pnts**) Let S be the surface of R, the region from the previous problem. Let $\boldsymbol{F} = \langle 2x, y, -z\rangle$. Find

$$\iint_S \boldsymbol{F}\cdot\boldsymbol{n}\,\mathrm{d}S$$

If you wish you may assume that the volume of R, call it V, is known (even if you could not answer the previous question).