

Instructions: Read all instructions carefully. Write your name, student number, and section number on your answer sheet. Clearly indicate your answers & show all your work; for many problems partial credit is available. 7 Problems worth 110 Points.

$$d\mathbf{R} = \mathbf{R}'(t)dt \quad d\mathbf{S} = \left( \frac{d\mathbf{R}}{dx} \times \frac{d\mathbf{R}}{dy} \right) dx dy \quad \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix}$$

**Multiple Choice; Write answer on your answer sheets; No partial credit.**

- P1 (5 pnts) Which of the following is a conservative vector field?  
 (a)  $\mathbf{F} = \langle x + z, 2y + x, z \rangle$     (b)  $\mathbf{F} = \langle y, x + z, 2z + y \rangle$     (c)  $\mathbf{F} = \langle 0, -z, 3y \rangle$ .

**Problems. Show all work on your answer sheets. Partial credit is available.**

- P2 (10 pnts) Clearly state the Divergence Theorem *or* Stokes' Theorem. You may write an equation for either theorem, but please state *what the equation means*.

- P3 (20 pnts) Let  $C$  be the curve extending from  $(-1, 1, 1)$  to  $(1, -1, 1)$ , and parametrized by  $\mathbf{R}(t) = \langle 2t - 1, \cos(\pi t), \cos(2\pi t) \rangle$ . Evaluate

$$\int_C \mathbf{F} \cdot d\mathbf{R},$$

where  $\mathbf{F}(x, y, z) = \frac{1}{\sqrt{x^2 + y^2 + z^2}} \langle x, y, z \rangle$ .

- P4 (20 pnts) Let  $S$  be the part of the cone  $x^2 = y^2 + z^2$  with  $2 \leq x \leq 3$ . Let the surface be oriented such that  $\mathbf{n} \cdot \mathbf{i} < 0$ , where  $\mathbf{n}$  is the surface unit normal. Let  $\mathbf{F} = \langle \frac{x}{2}, y, z \rangle$ . Evaluate

$$\iint_S \mathbf{F} \cdot d\mathbf{S}$$

- P5 (20 pnts) Let  $R$  be the region bounded by the  $y$  axis,  $y = \frac{1}{9}x^2 - 9$ , and  $y = 1 - x^2$ . Using the transformation  $x = uv, y = u^2 - v^2$ , rewrite the integral

$$\iint_R e^{y-x^2} dy dx$$

as an iterated integral in terms of  $u, v$ . Do *not* attempt to evaluate the integral. For full credit *you must write the limits of integration*. You may assume the transformed region  $\tilde{R}$  is in the first quadrant of  $u-v$  space.

- P6 (25 pnts) Let  $R$  be the region bounded by the inequalities:  $x^2 + \frac{1}{4}y^2 + \frac{1}{16}z^2 \leq 9$ , and  $x \geq 1$ . Find the volume of  $R$  *i.e.*, find

$$V = \iiint_R dx dy dz$$

- P7 (10 pnts) Let  $S$  be the surface of  $R$ , the region from the previous problem. Let  $\mathbf{F} = \langle 3x, 0, -z \rangle$ . Find

$$\iint_S \mathbf{F} \cdot \mathbf{n} dS$$

If you wish you may assume that the volume of  $R$ , call it  $V$ , is known (even if you could not answer the previous question).