

Instructions: Read all instructions carefully. Write your name, student number, and section number **on your answer sheet**. You will *not* hand in this sheet: clearly indicate your answers & show all your work on your answer sheet. For many problems partial credit is available. 10 Problems worth 160 Points.

Grading Notes: For those questions with multiple parts, please circle or box your answers so the grader does not have to hunt them down.

$$d\mathbf{R} = \mathbf{R}'(t)dt \quad d\mathbf{S} = \left(\frac{d\mathbf{R}}{dx} \times \frac{d\mathbf{R}}{dy} \right) dx dy \quad \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix}$$

$$\sin 2\theta = 2 \sin \theta \cos \theta \quad \cos 2\theta = \cos^2 \theta - \sin^2 \theta = 1 - 2 \sin^2 \theta = 2 \cos^2 \theta - 1$$

Problems. Show all work on your answer sheets. Partial credit is available.

- P1 (5 pnts) Define what a conservative vector field is. State two conditions equivalent to a field \mathbf{F} being conservative.
- P2 (10 pnts) State *both* the Divergence Theorem *and* Stokes' Theorem "in equation form." No explanation of the terms is necessary.
- P3 (15 pnts) Let S be the hemisphere $x^2 + y^2 + z^2 = 1$ above the xy plane, oriented such that $\mathbf{n} \cdot \mathbf{k} > 0$. Let $\mathbf{F} = \langle -z - y, x + y, z + x \rangle$. Evaluate

$$\iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$$

(Hint: There is a shortcut.)

- P4 (15 pnts) Consider the field $\mathbf{F}(x, y, z) = \langle y, -x, z \rangle$.
- (a) Is \mathbf{F} conservative? Why or why not?
- (b) Let C be the curve from $(4, 0, 0)$ to $(4, 0, 2\pi)$, parametrized by $\mathbf{R}(t) = \langle 4 \cos t, 4 \sin t, t \rangle$. Find

$$\int_C \mathbf{F} \cdot d\mathbf{R}$$

- P5 (15 pnts) The field $\mathbf{F}(x, y, z) = \langle y + 2xz, x, x^2 \rangle$ is known to be conservative.
- (a) Find the potential $\phi(x, y, z)$ for \mathbf{F} .
- (b) What is the curl of this field, *i.e.*, $\nabla \times \mathbf{F}$?
- (c) Let C be the curve parametrized by $\mathbf{R}(t) = \langle 1 + t^2, 1 + t^5, 1 + \cos(\pi t) \rangle$ for $0 \leq t \leq 1$. Find

$$\int_C \mathbf{F} \cdot d\mathbf{R}$$

Exam continues on reverse of page.

- P6 (20 pnts) Let R be the region bounded by the lines $y = x, y = x + 1, y = -x, y = -x + 1$. Evaluate

$$\iint_R (x^2 - y^2)^8 \, dA$$

by making the change of variables $u = x - y, v = x + y$.

- P7 (20 pnts) Given some number k , let $r_k(x, y, z)$ denote the function $(x^2 + y^2 + z^2)^k$. Let $\mathbf{R}(x, y, z) = \langle x, y, z \rangle$.

- Find the gradient ∇r_k .
- Find the divergence $\nabla \cdot \mathbf{R}$.
- Find the divergence $\nabla \cdot (r_k \mathbf{R})$.
- Find the curl $\nabla \times \mathbf{R}$.
- Find the curl $\nabla \times (r_k \mathbf{R})$.

- P8 (20 pnts) Let C be the closed curve parametrized by $\mathbf{R}(t) = \langle \sin t \cos t, \sin^2 t, 0 \rangle$ for $0 \leq t \leq \pi$. Let R be the region enclosed by C . Your job is to find the area of R .

- Find a field \mathbf{F} such that the area of R is (by Green's Theorem)

$$\oint_C \mathbf{F} \cdot d\mathbf{R}$$

- Find the area of R by evaluating the circulation of your field \mathbf{F} .

(Hint: It is to your advantage to select \mathbf{F} to be as simple as possible since you must evaluate the circulation in part (b).)

- P9 (20 pnts) Let S be the surface of the graph of $f(x, y) = 2xy + y^2$, for points (x, y) in the disc $x^2 + y^2 \leq 1$. Orient S such that $\mathbf{n} \cdot \mathbf{k} > 0$.

- Find the area element $d\mathbf{S}$ for S .
- Letting $\mathbf{F} = \langle x, -y, 2x^2 \rangle$, find the flux

$$\iint_S \mathbf{F} \cdot d\mathbf{S}$$

- P10 (20 pnts) Let R be the region bounded by the inequalities: $\frac{1}{4}x^2 + y^2 + \frac{1}{9}z^2 \leq 1$, and $y \geq 0$. Let S be the surface of R .

- Find the volume of R i.e., find

$$V = \iiint_R dx \, dy \, dz$$

- Let $\mathbf{F} = \langle y, x, \frac{z}{2} \rangle$. Find

$$\iint_S \mathbf{F} \cdot d\mathbf{S}$$