Instructions: Read all instructions carefully. Write your name, student number, and section number **on your answer sheet**. You will *not* hand in this sheet: clearly indicate your answers & show all your work on your answer sheet. For many problems partial credit is available. 10 Problems worth 160 Points.

Grading Notes: For those questions with multiple parts, please circle or box your answers so the grader does not have to hunt them down.

$$\mathrm{d}\boldsymbol{R} = \boldsymbol{R}'(t)\mathrm{d}t \quad \mathrm{d}\boldsymbol{S} = \left(\frac{\mathrm{d}\boldsymbol{R}}{\mathrm{d}x} \times \frac{\mathrm{d}\boldsymbol{R}}{\mathrm{d}y}\right)\mathrm{d}x\mathrm{d}y \quad \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix}$$

$$\sin 2\theta = 2\sin\theta\cos\theta \qquad \cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$$

Problems. Show all work on your answer sheets. Partial credit is available.

P1 (**5 pnts**) Define what a conservative vector field is. State two conditions equivalent to a field $\boldsymbol{F}$ being conservative.

P2 (**10 pnts**) State *both* the Divergence Theorem *and* Stokes' Theorem "in equation form." No explanation of the terms is necessary.

P3 (**15 pnts**) Let S be the hemisphere $x^2 + y^2 + z^2 = 4$ above the xy plane, oriented such that $\boldsymbol{n} \cdot \boldsymbol{k} > 0$. Let $\boldsymbol{F} = \langle -y, x - z, z - x \rangle$. Evaluate

$$\iint_S (\nabla \times \boldsymbol{F}) \cdot \mathrm{d}\boldsymbol{S}$$

(*Hint:* There is a shortcut.)

P4 (**15 pnts**) Consider the field $\boldsymbol{F}(x,y,z) = \langle -y, x, 2z \rangle$.

(a) Is $\boldsymbol{F}$ conservative? Why or why not?

(b) Let C be the curve from $(1,0,0)$ to $(1,0,4\pi)$, parametrized by $\boldsymbol{R}(t) = \langle \cos t, \sin t, 2t \rangle$. Find

$$\int_C \boldsymbol{F} \cdot \mathrm{d}\boldsymbol{R}$$

P5 (**15 pnts**) The field $\boldsymbol{F}(x,y,z) = \left\langle y^2, 2yx + z, y \right\rangle$ is known to be conservative.

(a) Find the potential $\phi(x,y,z)$ for $\boldsymbol{F}$.

(b) What is the curl of this field, *i.e.*, $\nabla \times \boldsymbol{F}$?

(c) Let C be the curve parametrized by $\boldsymbol{R}(t) = \left\langle 1 + t^3,\ 2 - t^5,\ 1 - \cos(\pi t) \right\rangle$ for $0 \le t \le 1$. Find

$$\int_C \boldsymbol{F} \cdot \mathrm{d}\boldsymbol{R}$$

Exam continues on reverse of page.

P6 (**20 pnts**) Let R be the region bounded by the lines $y=-x, y=-x+1, y=x, y=x+1$. Evaluate

$$\iint_R \left(x^2-y^2\right)^{10} \mathrm{d}A$$

by making the change of variables $u=x+y$, $v=x-y$.

P7 (**20 pnts**) Given some number k, let $r_k(x,y,z)$ denote the function $\left(x^2+y^2+z^2\right)^k$. Let $\boldsymbol{R}(x,y,z)=\langle x,y,z\rangle$.

(a) Find the gradient ∇r_k.
(b) Find the divergence $\nabla\cdot\boldsymbol{R}$.
(c) Find the divergence $\nabla\cdot(r_k\boldsymbol{R})$.
(d) Find the curl $\nabla\times\boldsymbol{R}$.
(e) Find the curl $\nabla\times(r_k\boldsymbol{R})$.

P8 (**20 pnts**) Let C be the closed curve parametrized by $\boldsymbol{R}(t)=\left\langle\cos^2 t,\cos t\sin t,0\right\rangle$ for $-\pi/2\le t\le\pi/2$. Let R be the region enclosed by C. Your job is to find the area of R.

(a) Find a field $\boldsymbol{F}$ such that the area of R is (by Green's Theorem)

$$\oint_C \boldsymbol{F}\cdot\mathrm{d}\boldsymbol{R}$$

(b) Find the area of R by evaluating the circulation of your field $\boldsymbol{F}$.

(*Hint:* It is to your advantage to select $\boldsymbol{F}$ to be as simple as possible since you must evaluate the circulation in part (b).)

P9 (**20 pnts**) Let S be the surface of the graph of $f(x,y)=x^2-xy$, for points (x,y) in the disc $x^2+y^2\le 4$. Orient S such that $\boldsymbol{n}\cdot\boldsymbol{k}>0$.

(a) Find the area element $\mathrm{d}\boldsymbol{S}$ for S.
(b) Letting $\boldsymbol{F}=\left\langle y,2y,x^2\right\rangle$, find the flux

$$\iint_S \boldsymbol{F}\cdot\mathrm{d}\boldsymbol{S}$$

P10 (**20 pnts**) Let R be the region bounded by the inequalities: $\frac{1}{16}x^2+\frac{1}{4}y^2+z^2\le 1$, and $z\ge 0$. Let S be the surface of R.

(a) Find the volume of R *i.e.*, find

$$V=\iiint_R \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$$

(b) Let $\boldsymbol{F}=\left\langle\frac{1}{4}x,z,x-\frac{y}{2}\right\rangle$. Find

$$\iint_S \boldsymbol{F}\cdot\mathrm{d}\boldsymbol{S}$$