

2005S M172 Homework 3

This homework is due May 2 in discussion section.

1. Do problem 3.2, 3.3 of the notes [1]. You may write your code in C, C++, Fortran, or octave/Matlab.
2. Consider the “operators” δ_x^+ , δ_x^- , δ_x , δ_{xx} , δ_t^+ , δ_t^- , δ_t , and δ_{tt} which operate on a doubly indexed variable U_n^j as follows:

$$\begin{aligned}
 \delta_x^+ U_n^j &= \frac{1}{h} (U_{n+1}^j - U_n^j) & \delta_t^+ U_n^j &= \frac{1}{k} (U_n^{j+1} - U_n^j) \\
 \delta_x^- U_n^j &= \frac{1}{h} (U_n^j - U_{n-1}^j) & \delta_t^- U_n^j &= \frac{1}{k} (U_n^j - U_n^{j-1}) \\
 \delta_x U_n^j &= \frac{1}{2h} (U_{n+1}^j - U_{n-1}^j) & \delta_t U_n^j &= \frac{1}{2k} (U_n^{j+1} - U_n^{j-1}) \\
 \delta_{xx} U_n^j &= \frac{1}{h^2} (U_{n+1}^j - 2U_n^j + U_{n-1}^j) & \delta_{tt} U_n^j &= \frac{1}{k^2} (U_n^{j+1} - 2U_n^j + U_n^{j-1})
 \end{aligned}$$

You are to find the “symbols” associated with each of these operators. That is, you are to find what they do to the Fourier component:

$$\mathcal{F}_n^j = \lambda^j e^{i\beta n h}.$$

I will do one as an example:

$$\begin{aligned}
 \delta_x^+ \mathcal{F}_n^j &= \delta_x^+ [\lambda^j e^{i\beta n h}] = \frac{1}{h} (\lambda^j e^{i\beta(n+1)h} - \lambda^j e^{i\beta n h}) \\
 &= \frac{\lambda^j}{h} e^{i\beta n h} (e^{i\beta h} - 1) = \mathcal{F}_n^j \frac{\cos(\beta h) + i \sin(\beta h) - 1}{h}
 \end{aligned}$$

Find the symbols for the remaining operators. In particular, you should find that:

$$\begin{aligned}
 \delta_x^+ \mathcal{F}_n^j &= \mathcal{F}_n^j \frac{\cos(\beta h) + i \sin(\beta h) - 1}{h} & \delta_t^+ \mathcal{F}_n^j &= \frac{\mathcal{F}_n^j}{k} (\lambda - 1) \\
 \delta_x \mathcal{F}_n^j &= \frac{\mathcal{F}_n^j}{h} i \sin(\beta h) & \delta_t \mathcal{F}_n^j &= \frac{\mathcal{F}_n^j}{2k} (\lambda - 1/\lambda) \\
 \delta_{xx} \mathcal{F}_n^j &= \frac{\mathcal{F}_n^j}{h^2} (-4 \sin^2(\beta h/2)) & \delta_{tt} \mathcal{F}_n^j &= \frac{\mathcal{F}_n^j}{k^2} \left(\lambda - 2 + \frac{1}{\lambda} \right)
 \end{aligned}$$

References

- [1] Randolph E. Bank, Peter Rentrop and Donald R. Smith. Numerical treatment of partial differential equations, 1995. Course Notes for UCSD Math M172.