2005S M20C Exam 1 Preparation

The first midterm exam is Wednesday April 20, during the class period. *You must bring a blue book to the exam.* Blue books are available from the bookstore. You also need to bring your student ID card or other form of ID (driver's license, passport, etc.) The exam covers material from §10.1, 10.2, 12.1-12.5, 13.1–13.3, 14.1.

The following formulæ will be provided on your exam:

$$\boldsymbol{a} \cdot \boldsymbol{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 \qquad \boldsymbol{a} \times \boldsymbol{b} = \langle a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1 \rangle$$

$$|\boldsymbol{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}.$$

Everything else must be committed to memory.

You should be prepared to answer at least the following questions:

1. What's the difference between a scalar and a vector? Describe the input and output types of common operations.

2. Let $P = (2, 3, 1)\,, Q = (4, -2, 5)$.

1. What is the "position vector" of P?
2. Find the distance from P to Q.
3. Find the vector from P to Q, $\boldsymbol{PQ}$.
4. Find the distance from P to the origin.
5. Find the distance from P to the xy plane.
6. Find the equation of the sphere centered at P with radius 5.
7. Find the equation of the sphere centered at Q such that P is on the sphere.

3. Let $\boldsymbol{a} = \langle 1, 0, 1 \rangle\,, \boldsymbol{b} = \langle 1, 3, 0 \rangle$.

1. Find $|\boldsymbol{a}|$.
2. Find a unit vector in the same direction as $\boldsymbol{a}$.
3. Find $\boldsymbol{a} + \boldsymbol{b}$.
4. Find $3\boldsymbol{a} - 2\boldsymbol{b}$.
5. Find $\boldsymbol{a} \cdot \boldsymbol{b}$.
6. Find $\boldsymbol{a} \times \boldsymbol{b}$.
7. Find a vector perpendicular to both $\boldsymbol{a}$ and $\boldsymbol{b}$.
8. Find the angle subtended by the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$.
9. Find the component of $\boldsymbol{a}$ along $\boldsymbol{b}$.
10. Find the projection of $\boldsymbol{a}$ onto $\boldsymbol{b}$.
11. Give a parametrization of the line through the point $(3, 4, 1)$ parallel to $\boldsymbol{b}$.
12. Give a parametrization of the plane through the origin that is parallel to both $\boldsymbol{a}$ and $\boldsymbol{b}$.
13. Give a parametrization of the plane containing the point $(1, 2, -1)$ that has $\boldsymbol{a}$ as a normal.
14. Find the angle subtended by $\boldsymbol{a}$ and $\boldsymbol{a} \times \boldsymbol{b}$.
15. Is $\boldsymbol{a}$ parallel to $\boldsymbol{b}$? Are they perpendicular?

16. Find a vector parallel to $\boldsymbol{a}$.

4. Let $\boldsymbol{u} = \langle 1, 2, -3 \rangle$, $\boldsymbol{v} = \langle -2, 4, 1 \rangle$, $\boldsymbol{w} = \langle -3, 0, 1 \rangle$.

1. Find $5\boldsymbol{u}$.
2. Find $\boldsymbol{u} \cdot \boldsymbol{u}$.
3. Find $|\boldsymbol{u}|$.
4. Find a unit length vector in the same direction as $\boldsymbol{u}$.
5. Find $|\alpha \boldsymbol{u}|$, where α is a real number.
6. Find $\boldsymbol{u} \cdot \boldsymbol{v}$.
7. Find the angle subtended by $\boldsymbol{u}$ and $\boldsymbol{v}$.
8. Find $\boldsymbol{v} \times \boldsymbol{w}$.
9. Find a vector perpendicular to both $\boldsymbol{v}$ and $\boldsymbol{w}$.
10. Find a unit length vector perpendicular to both $\boldsymbol{v}$ and $\boldsymbol{w}$.
11. Find $\boldsymbol{u} \times \boldsymbol{v}$.
12. Find $\boldsymbol{w} \cdot (\boldsymbol{u} \times \boldsymbol{v})$.
13. Find $\boldsymbol{u} \times \boldsymbol{u}$.

5. Let $\boldsymbol{R_1}(t) = \langle 0, 3, 1 \rangle + t \langle 2, 1, -1 \rangle$ parametrize the line l_1. Let $\boldsymbol{R_2}(t) = \langle 1, 2, 1 \rangle + t \langle 2, 3, 0 \rangle$ parametrize the line l_2. Let $3x - 2y + 5z = 1$ describe a plane P.

1. Find a vector parallel to l_1. Find a vector parallel to l_2.
2. Find a vector normal to P.
3. Do the lines l_1, l_2 intersect? If so, find their point of intersection. If not, are they parallel, or are they skew?
4. Find the intersection of l_1 and P.
5. Find the intersection of l_2 and P. (*Hint:* it's a trick)
6. Find the angle that l_1 subtends with the plane P.
7. Find the distance from l_1 to the point $(2, 2, 1)$.
8. Find the distance from l_2 to P.

6. Let $3x - 2y + 5z = 1$ describe a plane P. Let $x + y + 3z = 4$ describe a plane Q.

1. Find a vector normal to P.
2. Find a vector normal to Q.
3. Are the planes P, Q parallel? If not, must they intersect?
4. Find the angle subtended by the two planes

7. Let $\boldsymbol{u} = \langle -2, 1, 4 \rangle$, and let P be the point $(0, 4, 1)$.

1. Find a vector valued function $\boldsymbol{l}(t)$ which traces out the line through P and parallel to $\boldsymbol{u}$.
2. Find the equation of the plane containing P with normal $\boldsymbol{u}$.
3. Find the parametrization, $\boldsymbol{m}(t)$, of the line line through P and perpendicular to $\boldsymbol{u}$. (*Hint:* There are many answers. One way to do this is pick some arbitrary vector $\boldsymbol{v}$ not parallel to $\boldsymbol{u}$, then find a vector perpendicular to both $\boldsymbol{u}$ and $\boldsymbol{v}$.)

8. (§13.2) Find the limits:

1. $\lim_{t\to 0} \boldsymbol{R}(t)$, where $\boldsymbol{R}(t) = \left\langle t+1, 5, t^2 \right\rangle$.
2. $\lim_{t\to 0} \left\langle \frac{\sin t}{t}, e^{-t}, t \right\rangle$.
3. $\lim_{t\to 0} \left\langle \frac{t^2+t^4}{4t^2-t^3}, 5t+4, \ln(1+t) \right\rangle$.
4. $\lim_{t\to 1} \left\langle \frac{1-t}{1-t^2}, \frac{\ln t}{1-t}, 5t \right\rangle$.

9. (§13.2) Given $\boldsymbol{R}(t)$, find its antiderivative $\int \boldsymbol{R}(t)\,\mathrm{d}t$.

1. $\boldsymbol{R}(t) = \left\langle 2t-1, t^2, \cos t \right\rangle$.
2. $\boldsymbol{R}(t) = \left\langle \frac{1}{1+t^2}, \frac{1}{t}, \sqrt{2t+1} \right\rangle$.

10. (§13.2) Given vector valued functions $\boldsymbol{R}(t)$ and $\boldsymbol{S}(t)$, which describe the position of two particles, find whether the particles collide. Find if the paths of the two particles intersect.

1. $\boldsymbol{R}(t) = \left\langle 2t, t^2, \cos t \right\rangle$, $\boldsymbol{S}(t) = \left\langle \sin t, -e^t, 4t \right\rangle$.
2. $\boldsymbol{R}(t) = \langle 1+3t, 2-5t, -2+t \rangle$, $\boldsymbol{S}(t) = \langle 5-t, 1-4t, -3+2t \rangle$.
3. $\boldsymbol{R}(t) = \langle 1+t, 2+t, 3+t \rangle$, $\boldsymbol{S}(t) = \left\langle t-1, t^2, t+1 \right\rangle$.

11. (§13.2) Find $\boldsymbol{R}'(t)$ for

1. $\boldsymbol{R}(t) = \langle t, 3-2t, 4+6t \rangle$.
2. $\boldsymbol{R}(t) = \left\langle t+1, 5, t^2 \right\rangle$.
3. $\boldsymbol{R}(t) = \langle 4, 7, 1 \rangle \times \left\langle t+1, 5, t^2 \right\rangle$.
4. $\boldsymbol{R}(t) = \langle \arctan t, \sin t, \cos t \rangle$.
5. $\boldsymbol{R}(t) = \left\langle \arctan\left(e^t\right), \sin\left(e^t\right), \cos\left(e^t\right) \right\rangle$.
6. $\boldsymbol{R}(t) = \langle \cos t, \sin t, 3t \rangle$.
7. $\boldsymbol{R}(t) = \left\langle (t+1)\sin t, 5\sin t, t^2 \sin t \right\rangle$.

12. (§13.3) Given vector valued function $\boldsymbol{R}(t)$, find the arc length of the curve traced by $\boldsymbol{R}$ for t between t_0 and t_1, *e.g.*,

1. $\boldsymbol{R}(t) = \langle 1+3t, 3, 1-4t \rangle$, $t_0 = 0$, $t_1 = 3$.
2. $\boldsymbol{R}(t) = \langle 2\cos t, 2\sin t \rangle, 0 \le t \le \pi$.
3. $\boldsymbol{R}(t) = \langle \cos t, \sin t, 3t \rangle$, $t_0 = 0$, $t_1 = 4\pi$.
4. $\boldsymbol{R}(t) = \left\langle t/\sqrt{2}, t/\sqrt{2}, \ln(\sec t) \right\rangle$, $t_0 = 0$, $t_1 = \pi/3$.
5. $\boldsymbol{R}(t) = \left\langle t, t^3/3 + 1/(4t) \right\rangle$, $t_0 = 1$, $t_1 = 2$.
6. $\boldsymbol{R}(t) = \left\langle \sqrt{2}t, e^t, e^{-t} \right\rangle, 0 \le t \le 1$.
7. $\boldsymbol{R}(t) = \left\langle t^2, t, 3 \right\rangle, 0 \le t \le 1$.

13. (§13.4) Given the position of a particle, $\boldsymbol{R}(t)$, find its velocity, speed, and acceleration. *e.g.*,:

1. $\boldsymbol{R}(t) = \langle 2\cos t, 2\sin t \rangle$.
2. $\boldsymbol{R}(t) = \left\langle \sqrt{2}t, e^t, e^{-t} \right\rangle$.
3. $\boldsymbol{R}(t) = \left\langle t^2, t, 3 \right\rangle$.

14. (§13.4) Given the acceleration of a particle and its initial position and velocity, find the position of the particle. *e.g.*,:

1. $\boldsymbol{a}(t) = \langle 1, t, \sin t \rangle$, $\boldsymbol{v}(0) = \langle 0, 1, 0 \rangle$, $\boldsymbol{R}(0) = \langle 0, 0, 0 \rangle$.
2. $\boldsymbol{a}(t) = \langle \sin t, \cos t \rangle$, $\boldsymbol{v}(0) = \langle 1, 0 \rangle$, $\boldsymbol{R}(0) = \langle 0, 1 \rangle$.

15. (§14.1) You should have some basic understanding of functions of two or more variables. *e.g.,*: given $f(x, y)$, be able to evaluate $f(1, 2)$; find the domain of f; sketch a graph of $f(x, y)$, or the contours (level sets) of $f(x, y)$.

1. What is the domain of $f(x, y) = \frac{\sqrt{x-y}}{y}$?
2. Graph the level sets of $f(x, y) = \frac{1}{4}x^2 + y^2$.
3. Graph the function $f(x, y) = \sqrt{x^2 + y^2}$.
4. Graph the function and level sets of $f(x, y) = 3x - 4y + 2$.