Exam 1 v. 1		**F 2005 M20E : Vector Calculus**	
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Sec: A02 (9am)	A03 (10am)	A04 (11am)	A06 (1pm)

Instructions: Read all instructions carefully. Write your name, student number, and section *on your answer sheet.* Clearly indicate your answers & show all your work on your answer sheet. For many problems partial credit is available. For those questions with multiple parts, please circle or box your answers. 7 Problems worth 100 Points.

Hints: ∇ abbreviates $\boldsymbol{i}\frac{\partial}{\partial x}+\boldsymbol{j}\frac{\partial}{\partial y}+\boldsymbol{k}\frac{\partial}{\partial z}$

Multiple Choice; Write answer on your answer sheets; No partial credit.

P1 (**5 pnts**) Which of these is the equation of a plane with normal vector $\langle 1,-2,3\rangle$?
(a) $\boldsymbol{\ell}(t)=\langle 1+t,-2t,4+3t\rangle$ (b) $x-2y+3z+5=0$
(c) $\boldsymbol{R}(x,y,z)=\langle x,-2y,3z\rangle$ (d) none of the above.

P2 (**5 pnts**) Which of the following is the first order (*i.e.*, linear) Taylor approximation to $f(\boldsymbol{x_0}+\boldsymbol{h})$?
(a) $f(\boldsymbol{x_0}+\boldsymbol{h})\approx f(\boldsymbol{x_0})+\nabla f(\boldsymbol{x_0})\cdot\boldsymbol{h}$ (b) $f(\boldsymbol{x_0}+\boldsymbol{h})\approx f(\boldsymbol{x_0})+\boldsymbol{x_0}\cdot\boldsymbol{h}$
(c) $f(\boldsymbol{x_0}+\boldsymbol{h})\approx f(\boldsymbol{h})+\nabla f(\boldsymbol{x_0})\cdot\boldsymbol{h}$ (d) none of the above.

P3 (**6 pnts**) Which of the following is the Cartesian representation of the point whose Cylindrical coordinates are $r=2$, $\theta=\pi/6$, $z=2$?
(a) $(2,\pi/6,2)$ (b) $\left(2\sqrt{2},\pi/6,\pi/4\right)$
(c) $\left(1,\sqrt{3},2\right)$ (d) $\left(\sqrt{3},1,2\right)$ (e) none of the above.

P4 (**6 pnts**) Which of the following is a level set of $f(x,y)=x^2+y^2$?
(a) a straight line (b) a cone in space (c) a paraboloid
(d) a circle centered at the origin (e) none of the above

Problems. Show all work on your answer sheets. Partial credit is available.

P5 (**24 pnts**) Let $f(x,y,z)=xyz+x^2e^z$.
(a) Find ∇f.
(b) Evaluate ∇f at $(1,1,0)$.
(c) Find the directional derivative of f at $(1,1,0)$ along $\boldsymbol{v}=\langle -1,2,3\rangle$.
(d) Find the direction (*i.e.*, unit length vector) of maximal increase of f at the point $(1,1,0)$.

P6 (**24 pnts**) Let $\boldsymbol{F}(x,y,z)=\left\langle 2xy,2xy,x^2+y^2+z^2\right\rangle$.
(a) Find the divergence of $\boldsymbol{F}$.
(b) Find the curl of $\boldsymbol{F}$.
(c) Could $\boldsymbol{F}=\nabla\phi$ for some scalar field ϕ with continuous second derivatives?

P7 (**30 pnts**) Let $\boldsymbol{c}(t)=\left\langle 2t^{3/2},t,4t^{3/2}\right\rangle$.
(a) Find the velocity vector (as a function of t) of this path.
(b) Find the speed (as a function of t) of this path.
(c) Set up, *but do not attempt to solve*, a definite integral whose value is the arc length of this path for $1\le t\le 4$.
(d) Find a path $\boldsymbol{\ell}(\tau)$ which parameterizes the line tangent to $\boldsymbol{c}(t)$ at $t=1$.
(e) Let $F(x,y,z)=x^3+y^2-z$. Find $\frac{\mathrm{d}}{\mathrm{d}t}\left[F\circ\boldsymbol{c}(t)\right]$