Exam 1 v. 2		**F 2005 M20E : Vector Calculus**	
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Sec: A02 (9am)	A03 (10am)	A04 (11am)	A06 (1pm)

Instructions: Read all instructions carefully. Write your name, student number, and section *on your answer sheet.* Clearly indicate your answers & show all your work on your answer sheet. For many problems partial credit is available. For those questions with multiple parts, please circle or box your answers. 7 Problems worth 100 Points.

Hints: ∇ abbreviates $\boldsymbol{i}\frac{\partial}{\partial x} + \boldsymbol{j}\frac{\partial}{\partial y} + \boldsymbol{k}\frac{\partial}{\partial z}$

Multiple Choice; Write answer on your answer sheets; No partial credit.

P1 (**5 pnts**) Which of these is the equation of a plane with normal vector $\langle 2, -1, 1\rangle$?
(a) $2x - y + z + 5 = 0$ (b) $z = 2x - y + z$
(c) $\boldsymbol{\ell}(t) = \langle 2 + 2t, -1 - t, 1 + t\rangle$ (d) none of the above.

P2 (**5 pnts**) Which of the following is the first order (*i.e.*, linear) Taylor approximation to $f(\boldsymbol{x_0} + \boldsymbol{h})$?
(a) $f(\boldsymbol{x_0} + \boldsymbol{h}) \approx \nabla f(\boldsymbol{x_0}) + \boldsymbol{h}$ (b) $f(\boldsymbol{x_0} + \boldsymbol{h}) \approx f(\boldsymbol{x_0}) + \boldsymbol{x_0} \cdot \boldsymbol{h}$
(c) $f(\boldsymbol{x_0} + \boldsymbol{h}) \approx f(\boldsymbol{x_0}) + \nabla f(\boldsymbol{x_0}) \cdot \boldsymbol{h}$ (d) none of the above.

P3 (**6 pnts**) Which of the following is the Cartesian representation of the point whose Cylindrical coordinates are $r = 1$, $\theta = \pi/4$, $z = 4$?
(a) $\left(\sqrt{17}, \pi/4, \arctan 1/4\right)$ (b) $\left(\sqrt{2}, \sqrt{2}, \sqrt{2}\right)$ (c) $\left(\sqrt{2}/2, \sqrt{2}/2, 1\right)$
(d) $\left(-\sqrt{2}/2, -\sqrt{2}/2, 4\right)$ (e) none of the above.

P4 (**6 pnts**) Which of the following is a level set of $f(x, y) = 4x - y$?
(a) a plane in space (a) a straight line with slope 4 (c) a paraboloid
(d) an ellipse centered at the origin (e) none of the above

Problems. Show all work on your answer sheets. Partial credit is available.

P5 (**24 pnts**) Let $f(x, y, z) = xyz + y^2\sqrt{x}$.
(a) Find ∇f.
(b) Evaluate ∇f at $(1, -1, 1/2)$.
(c) Find the directional derivative of f at $(1, -1, 1/2)$ along $\boldsymbol{v} = \langle 1, 0, 3\rangle$.
(d) Find the direction (*i.e.*, unit length vector) of maximal increase of f at the point $(1, -1, 1/2)$.

P6 (**24 pnts**) Let $\boldsymbol{F}(x, y, z) = \left\langle z - x^2 - y^2, 2yz, 2yz\right\rangle$.
(a) Find the divergence of $\boldsymbol{F}$.
(b) Find the curl of $\boldsymbol{F}$.
(c) Could $\boldsymbol{F} = \nabla\phi$ for some scalar field ϕ with continuous second derivatives?

P7 (**30 pnts**) Let $\boldsymbol{c}(t) = \left\langle t, 2t, 2t^{3/2}\right\rangle$.
(a) Find the velocity vector (as a function of t) of this path.
(b) Find the speed (as a function of t) of this path.
(c) Set up, *but do not attempt to solve*, a definite integral whose value is the arc length of this path for $2 \le t \le 3$.
(d) Find a path $\boldsymbol{\ell}(\tau)$ which parameterizes the line tangent to $\boldsymbol{c}(t)$ at $t = 4$.
(e) Let $F(x, y, z) = x^2 + y^3 + z$. Find $\frac{\mathrm{d}}{\mathrm{d}t}\left[F \circ \boldsymbol{c}(t)\right]$