

<b>Exam 2 v. 1</b>	<b>F 2005 M20E : Vector Calculus</b>		
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<b>Sec:</b> A02 (9am)	A03 (10am)	A04 (11am)	A06 (1pm)

**Instructions:** Read all instructions carefully. Write your name, student number, and section *on your answer sheet*. Clearly indicate your answers & show all your work on your answer sheet. For many problems partial credit is available. For those questions with multiple parts, please circle or box your answers. 6 Problems worth 100 Points.

$$ds = c'(t) dt \quad d\mathbf{S} = (\mathbf{T}_u \times \mathbf{T}_v) du dv = \left( \frac{\partial \Phi}{\partial u} \times \frac{\partial \Phi}{\partial v} \right) du dv \quad \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix}$$

**Multiple Choice; Write answer on your answer sheets; No partial credit.**

P1 (5 pts) Which of the following is a conservative vector field?

- (a)  $\mathbf{F} = \langle xy, xy, 1 \rangle$       (b)  $\mathbf{F} = \langle 2y, 2x + 2z, 2y \rangle$       (c)  $\mathbf{F} = \langle 2xz^2, y, 1 \rangle$   
 (d)  $\mathbf{F} = \langle \cos x, \sin y, x \rangle$       (e)  $\mathbf{F} = \langle 4y + 4x, 4z + 4y, 4x + 4y \rangle$

**Problems. Show all work on your answer sheets. Partial credit is available.**

P2 (15 pts) Let  $f(x, y, z) = x^2 + y^2 + z^2$ . Let  $\mathbf{c}(t) = \langle 3 \cos t, 3 \sin t, t \rangle$ , be a path with  $0 \leq t \leq 2\pi$ . Evaluate the path integral of  $f$  along  $\mathbf{c}$ :

$$\int_{\mathbf{c}} f ds$$

P3 (20 pts) Let  $\mathbf{F}(x, y, z) = \langle \arctan(y^z), 4e^y - z, -x \rangle$ . Let  $\mathbf{c}(t) = \langle 1, t, e^t \rangle$ , for  $0 \leq t \leq \ln 4$ . Evaluate the line integral of  $\mathbf{F}$  along  $\mathbf{c}$ :

$$\int_{\mathbf{c}} \mathbf{F} \cdot d\mathbf{s}$$

P4 (20 pts) Let  $D$  be the region bounded by the curves  $y = 1/x$ ,  $y = 2/x$ ,  $x = 4$ ,  $x = 6$ . Rewrite the integral

$$\iint_D x^3 + y^2 x^3 dx dy$$

as an integral in  $u$  and  $v$  using the transformation  $(x, y) = T(u, v) = (u, v/u)$ . Do *not* attempt to solve this integral. For full credit *you must write the proper limits of integration* in  $u$  and  $v$ .

P5 (20 pts) Let  $S$  be the portion of the plane  $3x + 4y + 12z - 12 = 0$  with  $x \geq 0$ ,  $y \geq 0$ ,  $z \geq 0$ . Evaluate

$$\iint_S 48x dS$$

You may write your answer as the product of integers.

P6 (20 pts) Let  $\mathbf{F}(x, y, z) = \langle 2y, -x, 14 \cos^3(x^2 + y^2 - z^2) \rangle$ , and let  $S$  be the set  $\{(x, y, z) \mid x^2 + y^2 = 9, -1 \leq z \leq 1\}$ . Assume the normal of  $S$  points away from the  $z$ -axis. Evaluate

$$\iint_S \mathbf{F} \cdot d\mathbf{S}$$