

Exam 2 v. 2	F 2005 M20E : Vector Calculus		
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Sec: A02 (9am)	A03 (10am)	A04 (11am)	A06 (1pm)

Instructions: Read all instructions carefully. Write your name, student number, and section *on your answer sheet*. Clearly indicate your answers & show all your work on your answer sheet. For many problems partial credit is available. For those questions with multiple parts, please circle or box your answers. 6 Problems worth 100 Points.

$$ds = c'(t) dt \quad d\mathbf{S} = (\mathbf{T}_u \times \mathbf{T}_v) du dv = \left(\frac{\partial \Phi}{\partial u} \times \frac{\partial \Phi}{\partial v} \right) du dv \quad \frac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix}$$

Multiple Choice; Write answer on your answer sheets; No partial credit.

P1 (5 pts) Which of the following is a conservative vector field?

- (a) $\mathbf{F} = \langle -2x, -2z, 1 \rangle$ (b) $\mathbf{F} = \langle 4y + 4x, 4z + 4y, 4x + 4y \rangle$
(c) $\mathbf{F} = \langle \arctan x^y, e^{yz}, 0 \rangle$ (d) $\mathbf{F} = \langle y^2, z + 2yx, y \rangle$ (e) $\mathbf{F} = \langle 2, 2xz, 2xy \rangle$

Problems. Show all work on your answer sheets. Partial credit is available.

P2 (15 pts) Let $f(x, y, z) = x^2 + y^2 + e^z$. Let $\mathbf{c}(t) = \langle 2 \cos t, 2 \sin t, t \rangle$, be a path with $0 \leq t \leq \ln 2$. Evaluate the path integral of f along \mathbf{c} :

$$\int_{\mathbf{c}} f ds$$

P3 (20 pts) Let $\mathbf{F}(x, y, z) = \langle 3 \sin x + 2z, \arctan(x^z), y \rangle$. Let $\mathbf{c}(t) = \langle t, 3, \cos t \rangle$, for $0 \leq t \leq \pi/2$. Evaluate the line integral of \mathbf{F} along \mathbf{c} :

$$\int_{\mathbf{c}} \mathbf{F} \cdot d\mathbf{s}$$

P4 (20 pts) Let D be the region bounded by the curves $y = x/3$, $y = x/5$, $y = 2$, $y = 6$. Rewrite the integral

$$\iint_D \frac{1}{y} + \frac{x^2}{y^3} dx dy$$

as an integral in u and v using the transformation $(x, y) = T(u, v) = (uv, v)$. Do not attempt to solve this integral. For full credit you must write the proper limits of integration in u and v .

P5 (20 pts) Let S be the portion of the plane $3x + 5y + 8z - 15 = 0$ with $x \geq 0$, $y \geq 0$, $z \geq 0$. Evaluate

$$\iint_S \frac{\sqrt{2}}{7} x dS$$

You may write your answer as the ratio of two integers.

P6 (20 pts) Let $\mathbf{F}(x, y, z) = \langle -3y, 5x, 200 \tan(x^2 + y^2 + e^z) \rangle$, and let S be the set $\{(x, y, z) \mid x^2 + y^2 = 4, 0 \leq z \leq 3\}$. Assume the normal of S points away from the z -axis. Evaluate

$$\iint_S \mathbf{F} \cdot d\mathbf{S}$$