Final Exam v. 1		**F 2005 M20E : Vector Calculus**	
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Sec: A02 (9am)	A03 (10am)	A04 (11am)	A06 (1pm)

Instructions: Read all instructions carefully. Write your name, student number, and section *on your answer sheet.* Clearly indicate your answers & show all your work on your answer sheet. For many problems partial credit is available. For those questions with multiple parts, please circle or box your answers. 13 Problems worth 200 Points.

∇ abbreviates $\boldsymbol{i}\frac{\partial}{\partial x} + \boldsymbol{j}\frac{\partial}{\partial y} + \boldsymbol{k}\frac{\partial}{\partial z}$ $\quad \mathrm{d}\boldsymbol{s} = \boldsymbol{c}'(t)\,\mathrm{d}t$

$\mathrm{d}\boldsymbol{S} = \boldsymbol{T}_u \times \boldsymbol{T}_v \,\mathrm{d}u\,\mathrm{d}v = \left(\frac{\partial \boldsymbol{\Phi}}{\partial u} \times \frac{\partial \boldsymbol{\Phi}}{\partial v}\right) \mathrm{d}u\,\mathrm{d}v$ $\quad \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix}$

$\sin(2\theta) = 2\sin\theta\cos\theta$ $\quad \cos(2\theta) = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$

Multiple Choice; Write answer on your answer sheets; No partial credit.

P1 (**5 pnts**) Which of the following are the Cartesian coordinates of the point given in spherical coordinates by $(\rho, \phi, \theta) = (2, \pi/4, 3\pi/4)$?
(a) $\left(1, 1, \sqrt{2}\right)$ (b) $\left(1, 1, -\sqrt{2}\right)$ (c) $\left(1/2, \sqrt{3}/2, 1\right)$ (d) $\left(-1, 1, \sqrt{2}\right)$

P2 (**5 pnts**) A plane could be the level set of which one of the following functions?
(a) $f(x,y,z) = x^2 + y^2 + z^2$ (b) $f(x,y,z) = 3x - 2y + 4z$
(c) $f(x,y,z) = z^2 - x^2 + y^2$ (d) $f(x,y,z) = x + y^2 + z - 3$

P3 (**5 pnts**) Which one of the following vector fields is conservative?
(a) $\boldsymbol{F} = \langle xyz, xyz, xyz\rangle$ (b) $\boldsymbol{F} = \langle y+z, x+y, x+z\rangle$ (c) $\boldsymbol{F} = \langle -y, x, z\rangle$
(d) $\boldsymbol{F} = \left\langle 2xyz + y, x^2z + x, x^2y\right\rangle$ (e) $\boldsymbol{F} = \langle \cos xy, \sin xy, xy\rangle$

Problems. Show all work on your answer sheets. Partial credit is available.

P4 (**10 pnts**) State what it means for a vector field $\boldsymbol{F}$ to be conservative.

P5 (**10 pnts**) Let $\boldsymbol{F}(x,y,z) = \left\langle 2xy, x^2 + 2yz, y^2\right\rangle$. Let $\boldsymbol{c}(t) = \left\langle t, \sqrt{t}\cos(2\pi t), t^2 - 8\right\rangle$, be a path with $0 \le t \le 4$. Evaluate the line integral of $\boldsymbol{F}$ over $\boldsymbol{c}$:

$$\int_{\boldsymbol{c}} \boldsymbol{F} \cdot \mathrm{d}\boldsymbol{s}$$

P6 (**20 pnts**) State *both* the Divergence Theorem *and* Stokes' Theorem "in equation form." No explanation of the terms is necessary.

P7 (**20 pnts**) Let $\boldsymbol{F}(x,y,z) = \left\langle x^2 + y^2, yz, xy\right\rangle$.
(a) Find $\nabla \cdot \boldsymbol{F}$, the divergence of $\boldsymbol{F}$.
(b) Find $\nabla \times \boldsymbol{F}$, the curl of $\boldsymbol{F}$.

P8 (**20 pnts**) Let S be the surface $\left\{(x,y,z) \,\middle|\, z = x^2 + y^2,\ 0 \le z \le 4\right\}$, oriented such that $\boldsymbol{n} \cdot \boldsymbol{k} \le 0$. With $\boldsymbol{F} = \langle 6zx, 6zy, 2\rangle$, find

$$\iint_S \boldsymbol{F} \cdot \mathrm{d}\boldsymbol{S}$$

Exam continues on reverse of page.

P9 (**20 pnts**) Let $\phi(x, y, z) = xyz + e^{z+y-3x}$.

(a) Find $\nabla\phi$, the gradient of ϕ.

(b) Find the direction of maximal increase of ϕ at the point $(1, 3, 0)$. (Your answer should be a unit length vector.)

P10 (**20 pnts**) Let S be the region in $\mathbb{R}^2$ that is enclosed by the curve parametrized by $\boldsymbol{c}(t) = \langle 2\cos t, 5\sin t\rangle$ for $0 \le t \le 2\pi$.

(a) Give a field $\boldsymbol{F}$ such that, by Green's Theorem, the area of S is

$$\int_{\boldsymbol{c}} \boldsymbol{F} \cdot \mathrm{d}\boldsymbol{s}$$

(b) Find the area of S by evaluating the line integral for the field $\boldsymbol{F}$ you gave in the previous part.

P11 (**20 pnts**) Let $\boldsymbol{F} = \left\langle y, z^5, x\right\rangle$, and let S be the surface $\left\{(x, y, z) \,\middle|\, x^2 + y^2 + z^2 = 4,\ z \ge 0\right\}$, oriented such that $\boldsymbol{n} \cdot \boldsymbol{k} \ge 0$. Find

$$\iint_S (\nabla \times \boldsymbol{F}) \cdot \mathrm{d}\boldsymbol{S}$$

P12 (**20 pnts**) Let D be the region bounded by the curves $y = x^2$, $y = x^2/2$, $y = 4$, $y = 9$, with $x \ge 0$. Rewrite the integral

$$\iint_D \frac{x^5}{y^2} + x \,\mathrm{d}x\,\mathrm{d}y$$

as an integral in u and v using the transformation $(x, y) = T(u, v) = \left(v\sqrt{u}, v^2\right)$. Do *not* attempt to solve this integral. For full credit *you must write the proper limits of integration* in u and v.

P13 (**25 pnts**) Let W be the region $\left\{(x, y, z) \,\middle|\, x^2 + y^2 + z^2 \le 2,\ z \ge \sqrt{x^2 + y^2}\right\}$. Let S be the boundary of W, with outward pointing normal.

(a) Find V, the volume of W. That is, find

$$V = \iiint_W \mathrm{d}V$$

(b) Let $\boldsymbol{F} = \left\langle 2x + y^2, e^{3z}, xy - z\right\rangle$. Find

$$\iint_S \boldsymbol{F} \cdot \mathrm{d}\boldsymbol{S}$$